Math 220, Section 201
Homework #8

due Friday, March 22, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Suppose that $\lim_{x\to c+} f(x)$ and $\lim_{x\to c-} f(x)$ both exist but have different values. Prove that $\lim_{x\to c} f(x)$ does not exist.

II. Lay, p. 178, #20.4(a)

III. Lay, p. 178, #20.5(b)

IV. Let $P(x)$ be a polynomial. Prove that $\lim_{x\to c} P(x) = P(c)$ for every $c \in \mathbb{R}$. (You may assume that $\lim_{x\to c} x = c$ for every $c \in \mathbb{R}$.)

V. Write down a function $f(x)$ defined on $[0,2]$ such that $\lim_{x\to 1-} f(x) = 3$, $f(1) = 4$, and $\lim_{x\to 1+} f(x) = 5$.

March 22's quiz will be one of these five warm-up questions.

Homework Questions—hand these in

I. Lay, p. 178, #20.4(b)

II. Calculate each of the following limits (with justification) or explain why it doesn't exist.

(a) $\displaystyle\lim_{x\to 2} \frac{x^3+5}{x^2+2}$

(b) $\displaystyle\lim_{x\to 2} \frac{x^3-8}{x^2+2}$

(c) $\displaystyle\lim_{x\to 2} \frac{x^3+5}{x^2-4}$

(d) $\displaystyle\lim_{x\to 2} \frac{x^3-8}{x^2-4}$

(e) $\displaystyle\lim_{x\to 4+} \frac{\sqrt{x}-2}{x-4}$

(f) $\displaystyle\lim_{x\to 0} \frac{3x}{|x|}$

(g) $\displaystyle\lim_{x\to 0-} \frac{3x}{|x|}$

III. Let c be a real number, let D be an interval containing c, and let $f : D \to \mathbb{R}$ be a function on D.

(a) (Corollary 20.9) If $\lim_{x\to c} f(x)$ exists, prove that it is unique.

(b) (Theorem 20.10) Prove that $\lim_{x\to c} f(x)$ does not exist if and only if: there exists a sequence (s_n) with each $s_n \in D \setminus \{c\}$ such that (s_n) converges to c but $(f(s_n))$ is divergent. (Hint: Use Theorem 20.8 to help you. One implication should be easy.)

(continued on back of page)

IV. (a) Define a function $g : \mathbb{R} \to \mathbb{R}$ by

$$g(x) = \begin{cases} 1, & \text{if } x \in \mathbb{Q}; \\ 0, & \text{if } x \notin \mathbb{Q}. \end{cases}$$

Prove that $\lim_{x \to c} g(x)$ does not exist for any $c \in \mathbb{R}$.

(b) Define a function $h : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ by $h(x) = xg(x)$ for all $x \in \mathbb{R}$, $x \neq 0$. Find, with proof, $\lim_{x \to 0} h(x)$.