Math 220, Section 201
Homework #9
due Wednesday, April 3, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Lay, p. 187, #21.6

II. Lay, p. 187, #21.14

III. (a) Find a bounded set D and a continuous function $f : D \to \mathbb{R}$ such that f does not assume a minimum and/or a maximum value on D.
(b) Find a closed set E and a continuous function $g : E \to \mathbb{R}$ such that g does not assume a minimum and/or a maximum value on E.

IV. Lay, p. 193, #22.4

V. Lay, p. 193, #22.6

April 3's quiz will be one of these five warm-up questions.

Homework Questions—hand these in

I. Prove that the function

$$h(x) = \begin{cases} 5x, & \text{if } x \leq 2; \\ x^2 + 6, & \text{if } x > 2 \end{cases}$$

is continuous at $c = 2$.

II. Lay, p. 187, #21.16

III. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function, and let $V \subseteq \mathbb{R}$ be an open set. Define $U = \{x \in \mathbb{R} \colon f(x) \in V\}$. ($U$ is the *preimage of* V *under* f.) Prove that U is an open set.

IV. Let $f, g : \mathbb{R} \to \mathbb{R}$ be two functions. Suppose that f is bounded on $\mathbb{R}$ and $f(3) = 0$, and suppose that $\lim_{x \to 3} g(x) = 0$. Prove that the product function fg is continuous at $c = 3$.

V. Lay, p. 193, #22.8

VI. Assume that $\sin x$ and $\cos x$ are both continuous at 0. Prove that $\sin x$ and $\cos x$ are continuous at every real number c. (Hint: begin by writing $\sin x = \sin(c + y)$ where $y = x - c$ and using the addition formula for sin to expand $\sin(c + y)$.)

VII. A+ Question: Lay, p. 193, #22.10