Math 220, Section 203
Study Questions for Final Exam
(Friday, April 25, 2003)

I. D'Angelo and West, p. 288, #14.8

II. D'Angelo and West, p. 288, #14.9

III. D'Angelo and West, p. 288, #14.10

IV. D'Angelo and West, p. 288, #14.12

V. D'Angelo and West, p. 291, #14.59

VI. Let a and r be real numbers with $|r| < 1$. Prove that the series $\sum_{n=1}^{\infty} ar^n$ converges to $ar/(1-r)$.

VII. Prove that every convergent sequence is bounded.

VIII. (a) Define $s_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2}$. Prove that $s_n \leq 2 - \frac{1}{n}$ for every $n \in \mathbb{N}$.
(b) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.
(c) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^\pi}$ converges.

IX. (a) Define a sequence $\langle a \rangle = 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{16}, \ldots$; this sequence is given by the formula

$$a_n = \begin{cases} \frac{1}{2}, & \text{if } 1 \leq n < 2, \\ \frac{1}{4}, & \text{if } 2 \leq n < 4, \\ \frac{1}{8}, & \text{if } 4 \leq n < 8, \\ \frac{1}{16}, & \text{if } 8 \leq n < 16, \\ & \vdots \\ \frac{1}{2^k}, & \text{if } 2^{k-1} \leq n < 2^k, \\ & \vdots \end{cases}$$

Prove that $a_1 + a_2 + \cdots + a_{2^k - 1} = \frac{k}{2}$ for every positive integer k.
(b) Prove that the series $\sum_{n=1}^{\infty} a_n$ diverges.
(c) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.
(d) Let $p \leq 1$ be a real number. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges.