Math 220, Section 203—Homework #3
due in class Thursday, January 30, 2003

I. D'Angelo and West, p. 47, #2.28

II. D'Angelo and West, p. 47, #2.34(a)

III. D'Angelo and West, p. 47, #2.35. Note that the phrase "x and y are distinct real numbers" means "x and y are real numbers such that $x \neq y$".

IV. (a) D'Angelo and West, p. 48, #2.36
(b) State the converse of the statement you proved in part (a). Prove or disprove this converse.

V. D'Angelo and West, p. 48, #2.38

VI. We saw in class that phrasing the sentence "There is a unique real number such that P is true" in terms of symbolic logic was a little tricky: we had to write

$$(\exists x \in \mathbb{R})\Big(P(x) \wedge \big((\forall y \in \mathbb{R}, y \neq x)\big(\neg P(y)\big)\Big)$$

or equivalently

$$(\exists x \in \mathbb{R})\Big(P(x) \wedge \big((\forall y \in \mathbb{R})\big(P(y) \Rightarrow (y = x)\big)\Big).$$

In a similar way, phrase the following sentences in terms of symbolic logic:
(a) There are at least two real numbers such that P is true.
(b) There is at most one real number such that P is true.
(c) P is true for all real numbers with one exception.

VII. Follow the example of the Lewis Carroll *sorites* done in class to form a conclusion from the following *sorites*:
1. Animals that do not kick are never excitable.
2. Donkeys have no horns.
3. A buffalo can always toss you over a gate.
4. No animals that kick are easy to swallow.
5. No hornless animal can toss you over a gate.
6. All animals are excitable, except buffalos.