

**Math 220, Section 203—Homework #4**  
due in class Thursday, February 13, 2003

Remember that all of your solutions must be written in complete sentences that are easy to read and in logically correct order.

- I. D'Angelo and West, p. 24, #1.46
- II. D'Angelo and West, p. 24, #1.49
- III. D'Angelo and West, p. 24, #1.50
- IV. D'Angelo and West, p. 24, #1.51. (These sets are called *inverse images* and are discussed a bit further on page 14.)
- V. Determine which statements below are true. If true, provide a proof; if false, provide a counterexample.
  - (a) If  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  are increasing functions, then the function  $f + g$  is increasing.
  - (b) If  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  are increasing functions, then the function  $fg$  is increasing.
  - (c) If  $f : [0, \infty) \rightarrow \mathbb{R}$  and  $g : [0, \infty) \rightarrow \mathbb{R}$  are increasing functions, then the function  $fg$  is increasing.
- VI. Let  $H = \{x \in \mathbb{R} : 1/x \in \mathbb{N}\}$ . Prove that  $H \subseteq (0, 1]$  but  $H \not\subseteq [t, 1]$  for any positive real number  $t$ .
- VII.
  - (a) Let  $S_1 = \{x \in \mathbb{R} : x^n \leq x \text{ for every } n \in \mathbb{N}\}$ . Prove that  $S_1 = [0, 1]$ . (Don't neglect negative numbers  $x$ !)
  - (b) Let  $S_2 = \{x \in \mathbb{R} : x^n \geq x \text{ for every } n \in \mathbb{N}\}$ . Find (with proof) a simple expression for  $S_2$  in terms of intervals.
  - (c) Prove that for all real numbers  $x$ , if  $(x^{123} - x)(x^{124} - x) < 0$  then  $x < -1$ .