Math 220, Section 203—Homework #5
due in class Thursday, March 6, 2003

Remember that all of your solutions must be written in complete sentences that are easy to read and in logically correct order.

I. D'Angelo and West, pp. 72–73, #3.23 and #3.24

II. Find the flaw in the following "proof" that all horses are the same color:
"We prove, by induction on $n \in \mathbb{N}$, that in every group of n horses, all the horses are the same color. The base case, when $n = 1$, is obvious: in a group consisting of 1 single horse, clearly all horses in the group are the same color. For the induction step, assume that in every group of n horses, all horses are the same color; we need to prove that in every group of $n + 1$ horses, all horses are the same color. So let $\{h_1, \dots, h_{n+1}\}$ be a group of $n + 1$ horses. If we remove horse h_{n+1} temporarily, we are left with a group $\{h_1, \dots, h_n\}$ of n horses, all of which must be the same color by the induction hypothesis. Similarly, if we remove horse h_1 temporarily, we are left with a group $\{h_2, \dots, h_{n+1}\}$ of n horses, all of which must be the same color by the induction hypothesis. But since these two groups overlap, we conclude that all $n + 1$ horses in the full group are the same color, as desired."

III. D'Angelo and West, p. 74, #3.49(a),(b)

IV. Suppose that a_1, a_2, … are negative real numbers. Prove, using induction on n, that for all $n \in \mathbb{N}$, the number $(-1)^n a_1 \times \cdots \times a_n$ is positive.

V. D'Angelo and West, p. 75, #3.58(a). L-tilings are described in Solution 3.72 on pages 61–62 of D'Angelo and West.

VI. Imagine a robot that, whenever it sees a positive integer n written down, subtracts the number of digits of n from n and writes the answer down. For instance, if the robot saw the integer 108 written down, it would subtract 3 (since 108 has three digits) and write the number 105 down. Suppose you could tell the robot to keep doing this with all the numbers it writes down. For example, after writing down 105, the robot would write down 102 ($= 105 - 3$), then 99 ($= 102 - 3$), then 97 ($= 99 - 2$, since 99 has two digits), and so on. Prove that no matter what positive integer n the robot first sees, eventually the robot will see the the number 1 written down.