Math 220, Section 203—Homework #8
due in class Thursday, April 3, 2003

Remember that all of your solutions must be written in complete sentences that are easy to read and in logically correct order.

I. D'Angelo and West, p. 269, #13.25

II. D'Angelo and West, p. 269, #13.28

III. D'Angelo and West, p. 288, #14.18

IV. D'Angelo and West, p. 288, #14.21 (Hint: one way to solve this problem is a proof by contradiction, using the Completeness Axiom.)

V. Prove rigorously that the sequence $2, 5, 8, 11, 14, \ldots$ diverges. (Hint: start by finding a formula for the nth term of the sequence.)

VI. Let c and d be real numbers with $d > 0$. Prove that $\frac{c}{n^d} \to 0$.

VII. Suppose $a_1, a_2, \ldots$ is a sequence that converges to 1, and $b_1, b_2, \ldots$ is a sequence that converges to 2. Define a new sequence $\langle c \rangle$ by

$$c_n = \begin{cases} a_k, & \text{if } n = 2k-1 \text{ is odd,} \\ b_k, & \text{if } n = 2k \text{ is even.} \end{cases}$$

(In other words, $\langle c \rangle = a_1, b_1, a_2, b_2, \ldots$.) Prove that $\langle c \rangle$ does not converge.

Note: during class I wasn't able to find the reference in the textbook to the fact that every constant sequence $c, c, c, \ldots$ converges to c. This fact is proved in part 2 of Example 13.11 on page 259 of D'Angelo and West.