

Math 220, Section 203
Study Questions for First Midterm
(in class Tuesday, February 4, 2003)

- I. D'Angelo and West, p. 21, #1.7
- II. D'Angelo and West, p. 21, #1.13
- III. D'Angelo and West, p. 21, #1.14
- IV. D'Angelo and West, p. 21, #1.15
- V. D'Angelo and West, p. 22, #1.21
- VI. D'Angelo and West, p. 22, #1.27
- VII. D'Angelo and West, p. 23, #1.34
- VIII. D'Angelo and West, p. 23, #1.40 (only the first part, not the part about the U.S. states)
- IX. D'Angelo and West, p. 46, #2.24
- X. D'Angelo and West, p. 49, #2.49
- XI. Using the logical symbols we have been working with, find a formula that expresses the "exclusive or". That is, find a formula $F(P, Q)$ involving the sentences P and Q that has the following truth table:

P	Q	$F(P, Q)$
T	T	F
T	F	T
F	T	T
F	F	F

- XII. Prove that $\mathbb{Z} \cap (\frac{1}{4}, \frac{3}{4}) = \emptyset$.
- XIII. Find set identities that are analogous to each of the following tautologies:
 - (a) $(P \wedge Q) \Rightarrow P$
 - (b) $(P \wedge \neg P) \Rightarrow Q$
 - (c) $(P \wedge (Q \vee R)) \Leftrightarrow ((P \wedge Q) \vee (P \wedge R))$
 - (d) $P \vee (\neg P)$
- XIV. In class, we learned that to prove $A = \emptyset$, we should show that

$$(\forall x)(x \notin \emptyset).$$

On the other hand, we learned that to prove that any two sets A and B are equal, we need to prove

$$(\forall x)((x \in A \Rightarrow x \in B) \wedge (x \in B \Rightarrow x \in A)).$$

It seems we should be able to take $B = \emptyset$, so that proving that $A = \emptyset$ requires proving

$$(\forall x)((x \in A \Rightarrow x \in \emptyset) \wedge (x \in \emptyset \Rightarrow x \in A)).$$

Is this logically equivalent to what we learned in class?