I. All three parts of this problem ask you to determine whether certain statements are true. If a statement is true, simply say so in this problem; if it is false, give a counterexample.

   (a) D'Angelo and West, p. 48, #2.37

   (b) For each of the statements (a)–(g) listed in #2.37 in the book, write the converse (in symbolic notation). Which of these converse statements are true for all $x \in \mathbb{R}$?

   (c) For each of the statements (a)–(g) listed in #2.37 in the book, write the contrapositive (in symbolic notation). Which of these contrapositive statements are true for all $x \in \mathbb{R}$?

II. D'Angelo and West, p. 48, #2.44

III. (a) Find a statement $P(x)$ such that $(\exists x \in \mathbb{Q})(P(x))$ is true but $(\exists x \in \mathbb{Z})(P(x))$ is false. Justify your answer.

   (b) Find a statement $Q(x)$ such that $(\forall x \in \mathbb{Q})(Q(x))$ is true but $(\forall x \in \mathbb{R})(Q(x))$ is false. Justify your answer.

   (c) Can you find a statement $R(x)$ such that $(\exists x \in \mathbb{Z})(R(x))$ is true but $(\exists x \in \mathbb{Q})(R(x))$ is false? Can you find a statement $R(x)$ such that $(\forall x \in \mathbb{R})(R(x))$ is true but $(\forall x \in \mathbb{Q})(R(x))$ is false? Why or why not?

IV. Write three implications with different conclusions, all of which are equivalent to the implication "If your goldfish watches TV and listens to the radio and eats every day, then it is happy."

V. (a) Prove that the statements $(P \wedge Q) \Rightarrow R$ and $(P \Rightarrow R) \vee (Q \Rightarrow R)$ are logically equivalent using a chain of symbolic equivalences. (Hint: one way uses the distributive laws of symbolic logic.)

   (b) Suppose we let

   $$P = \text{"Fred is outside while it's raining"}$$
   $$Q = \text{"Fred isn't holding up his umbrella"}$$
   $$R = \text{"Fred is getting wet"}$$

   Then it seems like $(P \wedge Q) \Rightarrow R$ is true but $(P \Rightarrow R) \vee (Q \Rightarrow R)$ is false, contrary to part (a)! Explain why there isn't any contradiction.