Math 220, Section 201/202–Homework #3

due at the beginning of class Wednesday, January 26, 2005

I. D'Angelo and West, p. 47, #2.28

II. D'Angelo and West, p. 47, #2.34(b)

III. D'Angelo and West, p. 47, #2.35. Note that the phrase "x and y are distinct real numbers" means "x and y are real numbers such that $x \neq y$".

IV. (a) Let x be a real number. Prove that if $|x - 1| < 2$ then $|x^2 + 2x + 2| < 17$.
(b) State the converse of the statement you proved in part (a). Prove or disprove this converse.

V. D'Angelo and West, p. 48, #2.38

VI. The sentence "There is a unique real number such that P is true" can be rephrased in a more "proof-oriented" way as: "There is a real number x such that $P(x)$ is true and, for every real number y distinct from x, $P(y)$ is false."

In a similar manner, phrase the following sentences in more "proof-oriented" ways:

(a) There are at least two real numbers such that P is true.
(b) There is at most one real number such that P is true.
(c) P is true for all real numbers with one exception.

VII. Follow the example of the Lewis Carroll *sorites* done in class to form a conclusion from the following *sorites*:

1. I trust every animal that belongs to me.
2. Dogs gnaw bones.
3. I admit no animals into my study, unless they will beg when told to do so.
4. All the animals in the yard are mine.
5. I admit every animal, that I trust, into my study.
6. The only animals, that are really willing to beg when told to do so, are dogs.