Math 220, Section 201/202—Homework #4
due in class Wednesday, February 9, 2005

Remember that all of your solutions must be written in complete sentences that are easy to read and in logically correct order.

I. D'Angelo and West, p. 24, #1.50

II. Let $H = \{x \in \mathbb{R} \colon 1/x \in \mathbb{N}\} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\}$. Prove that $H \subseteq (0, 1]$ but $H \not\subseteq [t, 1]$ for any positive real number t.

III. (a) D'Angelo and West, p. 72, #3.23
(b) D'Angelo and West, p. 73, #3.24
(c) Find the flaw in the following "proof" that all horses are the same color:
"We prove, by induction on $n \in \mathbb{N}$, that in every group of n horses, all the horses are the same color. The base case, when $n = 1$, is obvious: in a group consisting of 1 single horse, clearly all horses in the group are the same color. For the induction step, assume that in every group of n horses, all horses are the same color; we need to prove that in every group of $n + 1$ horses, all horses are the same color. So let $\{h_1, \dots, h_{n+1}\}$ be a group of $n + 1$ horses. If we remove horse h_{n+1} temporarily, we are left with a group $\{h_1, \dots, h_n\}$ of n horses, all of which must be the same color by the induction hypothesis. Similarly, if we remove horse h_1 temporarily, we are left with a group $\{h_2, \dots, h_{n+1}\}$ of n horses, all of which must be the same color by the induction hypothesis. But since these two groups overlap, we conclude that all $n + 1$ horses in the full group are the same color, as desired."

IV. D'Angelo and West, p. 74, #3.49(c),(d)

V. Let a_n be the number of dots in a hexagonal arrangement consisting of n rings of dots, as illustrated in D'Angelo and West, p. 73, #3.39 for $n = 1$, 2, and 3. Find, with proof, a formula for a_n.

VI. Consider a chocolate bar that is made up of many little squares of chocolate attached to each other in a grid, say a grid with m rows and n columns (where m and n are positive integers). If our goal is to break the whole bar into its individual squares, we can proceed as follows: first, break the bar into two pieces along any of the horizontal or vertical lines separating the squares. Then, choose one of the resulting pieces and break it in two along one of its horizontal or vertical lines. Continue in this manner, at each step choosing one of the existing pieces and breaking it in two (never breaking multiple pieces at the same time). Using strong induction, prove that no matter how you choose the break to make at each step, the total number of breaks to get all of the little squares separated from one another is exactly $mn - 1$.