Math 220–Homework #7

due at the beginning of class Wednesday, March 16, 2005

- I. D'Angelo and West, p. 269, #13.25
- II. D'Angelo and West, p. 269, #13.28
- III. D'Angelo and West, p. 288, #14.21
- IV. (a) Let {a_n} be a convergent sequence. Prove that {a_n} is bounded.
 (b) Let {b_n} be an unbounded sequence. Prove that {b_n} is divergent.
- V. Suppose that $\{c_n\}$ is a sequence that converges to the real number $M \neq 0$. Suppose further that $c_n \neq 0$ for all $n \in \mathbb{N}$. Prove directly—that is, without using Theorem 14.5(c)—that the sequence $\{1/c_n\}$ converges to 1/M.
- VI. Suppose $\{a_n\}$ is a sequence that converges to 1 and $\{b_n\}$ is a sequence that converges to 2. Define a new sequence $\{c_n\}$ by

$$c_n = \begin{cases} a_k, & \text{if } n = 2k - 1 \text{ is odd,} \\ b_k, & \text{if } n = 2k \text{ is even.} \end{cases}$$

(In other words, $\{c_n\} = \{a_1, b_1, a_2, b_2, \dots\}$.) Prove that $\{c_n\}$ diverges.