Math 220, Section 201/202
Study Questions for First Midterm

I. D'Angelo and West, p. 21, #1.7

II. D'Angelo and West, p. 21, #1.13

III. D'Angelo and West, p. 21, #1.14

IV. D'Angelo and West, p. 21, #1.15

V. D'Angelo and West, p. 22, #1.21

VI. D'Angelo and West, p. 22, #1.27

VII. D'Angelo and West, p. 23, #1.34

VIII. D'Angelo and West, p. 23, #1.40 (only the first part, not the part about the U.S. states)

IX. D'Angelo and West, p. 46, #2.24

X. D'Angelo and West, p. 49, #2.49

XI. Using the logical symbols we have been working with, find a formula that expresses the "exclusive or". That is, find a formula $F(P, Q)$ involving the sentences P and Q that has the following truth table:

P	Q	$F(P, Q)$
T	T	F
T	F	T
F	T	T
F	F	F

XII. Prove that $\mathbb{Z} \cap (\frac{1}{4}, \frac{3}{4}) = \emptyset$.

XIII. Find set identities that are analogous to each of the following tautologies:
(a) $(P \wedge Q) \Rightarrow P$
(b) $(P \wedge \neg P) \Rightarrow Q$
(c) $\big(P \wedge (Q \vee R)\big) \Leftrightarrow \big((P \wedge Q) \vee (P \wedge R)\big)$
(d) $P \vee (\neg P)$

XIV. To prove $A = \emptyset$, we should show that

$$(\forall x)(x \notin \emptyset).$$

On the other hand, we learned that to prove that any two sets A and B are equal, we need to prove

$$(\forall x)\big((x \in A \Rightarrow x \in B) \wedge (x \in B \Rightarrow x \in A)\big).$$

It seems we should be able to take $B = \emptyset$, so that proving that $A = \emptyset$ requires proving

$$(\forall x)\big((x \in A \Rightarrow x \in \emptyset) \wedge (x \in \emptyset \Rightarrow x \in A)\big).$$

Is this logically equivalent to the first formulation?

XV. D'Angelo and West, p. 24, #1.46

XVI. D'Angelo and West, p. 24, #1.49

XVII. D'Angelo and West, p. 24, #1.51. (These sets are called *inverse images* and are discussed a bit further on page 14.)

XVIII. Determine which statements below are true. If true, provide a proof; if false, provide a counterexample.

(a) If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are increasing functions, then the function $f + g$ is increasing.

(b) If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are increasing functions, then the function fg is increasing.

(c) If $f : [0, \infty) \to \mathbb{R}$ and $g : [0, \infty) \to \mathbb{R}$ are increasing functions, then the function fg is increasing.

XIX. (a) Let $S_1 = \{x \in \mathbb{R} \colon x^n \leq x \text{ for every } n \in \mathbb{N}\}$. Prove that $S_1 = [0, 1]$. (Don't neglect negative numbers x!)

(b) Let $S_2 = \{x \in \mathbb{R} \colon x^n \geq x \text{ for every } n \in \mathbb{N}\}$. Find (with proof) a simple expression for S_2 in terms of intervals.

(c) Prove that for all real numbers x, if $(x^{123} - x)(x^{124} - x) < 0$ then $x < -1$.