Math 223, Section 101—Homework #1

due Wednesday, September 15, 2010 at the beginning of class

Practice problems. Don't turn in these problems. Most of these problems are easy, and most of them have answers in the back of the book, so they can be useful to you to check your understanding of the material.

1. Friedberg, Insel, and Spence, Section 1.2, pp. 12–13, #1(a)–(d),(f),(h)–(j)
2. Friedberg, Insel, and Spence, Section 1.2, p. 13, #3 and #4(a),(c),(e),(g)
3. Friedberg, Insel, and Spence, Section 1.2, p. 15, #11 and #13
4. Friedberg, Insel, and Spence, Section 1.3, pp. 19–20, #1(b),(c),(e),(f)
5. Friedberg, Insel, and Spence, Section 1.3, p. 20, #2(a),(c),(e),(g), #3, #4, and #8(a),(c),(e)
6. Friedberg, Insel, and Spence, Section 1.3, p. 21, #15
7. Friedberg, Insel, and Spence, Section 1.4, p. 32, #1(a)–(c)

Homework problems. Write up and hand in solutions to these problems. Please have your solutions to these problems in the correct order on your pages. Write clearly and legibly, in complete sentences; your solutions will be better if you think about how to best phrase your answer before you begin to write it. Please staple the pages together (unstapled solutions will not be accepted).

1. Friedberg, Insel, and Spence, Section 1.2, pp. 15–16, #18 and #19
2. Let $V = \{(x,y)\colon x > 0,\ y > 0\}$ be the set of all ordered pairs of positive real numbers. Define vector addition on V by the formula
$$(x_1, y_1) + (x_2, y_2) = (x_1x_2, y_1y_2)$$
and define scalar multiplication on V by the formula
$$c(x,y) = (x^c, y^c).$$
Prove that with this vector addition and scalar multiplication, V is a vector space.
3. A function $f \in \mathcal{F}(\mathbb{R},\mathbb{R})$ is called an *even* function if $f(-t) = f(t)$ for every real number t. A function $g \in \mathcal{F}(\mathbb{R},\mathbb{R})$ is called an *odd* function if $g(-t) = -g(t)$ for every real number t. Prove that the set of even functions and the set of odd functions are both subspaces of $\mathcal{F}(\mathbb{R},\mathbb{R})$.
4. Friedberg, Insel, and Spence, Section 1.3, p. 20, #5. You can use, without proof, the statements in Friedberg, Insel, and Spence, Section 1.3, p. 20, #3 and #4 if you wish.
5. Friedberg, Insel, and Spence, Section 1.3, p. 20, #8(b),(d),(f)
6. Friedberg, Insel, and Spence, Section 1.3, p. 21, #17 and #20. In #17, where the question says "$a \in F$", you should replace that by "$a \in \mathbb{R}$".
7. Let V be a vector space, and let $x \neq \underline{0}$ be a nonzero vector in V. Prove that $\operatorname{span}(\{x\}) = \{ax\colon a \in \mathbb{R}\}$.
8. Friedberg, Insel, and Spence, Section 1.4, p. 35, #15

Bonus question. Friedberg, Insel, and Spence, Section 1.3, p. 21, #19