Math 223, Section 101-Homework \#10
due Friday, December 3, 2010 at the beginning of class
Practice problems. Don't turn in these problems. Most of these problems are easy, and most of them have answers in the back of the book, so they can be useful to you to check your understanding of the material.

1. Friedberg, Insel, and Spence, Section 6.5, p. 392, \#1(a)-(h)
2. Friedberg, Insel, and Spence, Section 6.5, p. 392, \#2(a),(d) and \#3
3. Friedberg, Insel, and Spence, Section 6.5, p. 393, \#4 and \#5
4. Friedberg, Insel, and Spence, Section 6.5, p. 394, \#17

Homework problems. Write up and hand in solutions to these problems. Please have your solutions to these problems in the correct order on your pages. Write clearly and legibly, in complete sentences; your solutions will be better if you think about how to best phrase your answer before you begin to write it. Please staple the pages together (unstapled solutions will not be accepted).

1. Friedberg, Insel, and Spence, Section 6.5, p. 392, \#2(c),(e)
2. (a) Friedberg, Insel, and Spence, Section 6.4, p. 376, \#10. Hint: when can $T(x)-i x$ equal $\underline{0}$ ?
(b) Friedberg, Insel, and Spence, Section 6.5, p. 393, \#8
3. Friedberg, Insel, and Spence, Section 6.5, p. 394, \#11
4. Let $A=\left(\begin{array}{ll}3 & 2 \\ 2 & 0\end{array}\right)$.
(a) Find an orthogonal matrix $Q$ such that $Q^{-1} A Q$ is a diagonal matrix.
(b) Find eight distinct orthogonal matrices $Q$ such that $Q^{-1} A Q$ is a diagonal matrix. (You can include the one from part (a).)
5. Let $T$ and $U$ be two self-adjoint operators on a finite-dimensional inner product space $V$.
(a) Prove that there exists an orthonormal basis $\beta$ of $V$ consisting of eigenvectors for $T-U$.
(b) Suppose that $\langle x, T(x)\rangle=\langle x, U(x)\rangle$ for all $x \in V$. Prove that $T=U$. (Hint: consider $\langle v,(T-U)(v)\rangle$ for $v \in \beta$, where $\beta$ is from part (a).)

Bonus question. Let $V$ be a complex vector space, let $W$ be a complex inner product space, and let $T, U$ be two linear transformations from $V$ to $W$. Suppose that $\langle T(x), U(x)\rangle=0$ for all $x \in V$.
(a) Show that $R(T) \subseteq R(U)^{\perp}$ and $R(U) \subseteq R(T)^{\perp}$. (Hint: consider $\langle T(x+y), U(x+y)\rangle$ and $\langle T(x+i y), U(x+i y)\rangle$.
(b) Suppose that $x$ and $y$ are vectors in $V$ such that $T(x)=U(y)$. Show that $x \in N(T)$ and $y \in N(U)$.
This is the last homework of the course-congratulations on your hard work!

