Math 223, Section 101—Homework #2

due Wednesday, September 22, 2010 at the beginning of class

Practice problems. Don't turn in these problems. Most of these problems are easy, and most of them have answers in the back of the book, so they can be useful to you to check your understanding of the material.

- 1. Friedberg, Insel, and Spence, Section 1.4, p. 33, #2(a),(c),(e), #3(a),(c),(e), and #4(a),(c),(e)
- 2. Friedberg, Insel, and Spence, Section 1.4, p. 34, #5(a),(c),(e),(g)
- 3. Friedberg, Insel, and Spence, Section 1.5, p. 40, #1(a),(b),(d)–(f)
- 4. Friedberg, Insel, and Spence, Section 1.5, pp. 40–41, #2(a),(c),(e),(g),(i)
- 5. Friedberg, Insel, and Spence, Section 1.5, p. 41, #5 and #7. For #7, there are many possible answers; come up with at least one answer different from the one in the back of the book.
- 6. Friedberg, Insel, and Spence, Section 1.5, p. 42, #13 and #18
- 7. Friedberg, Insel, and Spence, Section 3.4, pp. 194–195, #1(d),(e)
- 8. Friedberg, Insel, and Spence, Section 3.4, pp. 195–196, #2(a),(c),(e),(g),(i)
- 9. Let x and y be nonzero vectors in a vector space V, and let a and b be nonzero real numbers. Show that:
 - (a) if ax = ay, then x = y.
 - (b) if ax = bx, then a = b. In other words, all scalar multiples of a nonzero vector x are different from one another.

Hint: use part "(bonus d)" of Theorem 1.2, stated in class.

Homework problems. Write up and hand in solutions to these problems. Please have your solutions to these problems in the correct order on your pages. Write clearly and legibly, in complete sentences; your solutions will be better if you think about how to best phrase your answer before you begin to write it. Please staple the pages together (unstapled solutions will not be accepted).

- 1. Let V be the set of all infinite *sequences* of real numbers; a typical element of V looks like $\{a_1, a_2, a_3, \ldots\}$. With addition and scalar multiplication defined componentwise, V is a vector space (you don't have to prove this).
 - (a) Let W be the set of all *convergent* sequences, that is, the set of all sequences $\{a_1, a_2, a_3, ...\}$ such that $\lim_{n\to\infty} a_n$ exists. Prove that W is a subspace of V.
 - (b) Let X be the set of all *eventually zero* sequences; typical elements of W are $\{3, 2, 1, 0, 0, 0, ...\}$ and $\{0, \pi, 0, e, -\sqrt{2}, 42, 0, 0, 0, ...\}$, where all remaining elements equal 0. Prove that X is a subspace of W.
- 2. Friedberg, Insel, and Spence, Section 3.4, p. 195, #2(b),(d),(f). Don't be afraid if you get double-digit denominators in (d).
- 3. Friedberg, Insel, and Spence, Section 1.4, p. 34, #5(d),(f),(h)
- 4. Friedberg, Insel, and Spence, Section 1.5, pp. 40-41, #2(d),(f),(h)
- 5. Friedberg, Insel, and Spence, Section 1.5, p. 41, #9 and #10

- 6. Friedberg, Insel, and Spence, Section 1.5, p. 42, #12
- 7. Let n be a positive integer.
 - (a) Prove that the polynomials $1, x, x^2, \ldots, x^n$ generate $P_n(\mathbb{R})$. (Yes, this is easy.)
 - (b) Prove that the polynomials $1, x + 19, (x + 19)^2, \dots, (x + 19)^n$ generate $P_n(\mathbb{R})$. (Is there a way to use part (a) to help?)
- 8. Let r_1, \ldots, r_n be distinct real numbers, and define the following functions in $\mathcal{F}(\mathbb{R}, \mathbb{R})$:

$$f_1(t) = e^{r_1 t}, \quad f_2(t) = e^{r_2 t}, \quad \dots, \quad f_n(t) = e^{r_n t}.$$

Show that $\{f_1(t), \ldots, f_n(t)\}$ is a linearly independent set. (Suggestion: use your knowledge of calculus, and consider what happens as $t \to \infty$.)

Bonus question. Let r_1, \ldots, r_n be distinct real numbers. Prove that for every polynomial $p(x) \in P_{n-1}(\mathbb{R})$, there is a unique set of real numbers a_1, \ldots, a_n such that

$$\frac{p(x)}{(x-r_1)\cdots(x-r_n)} = \frac{a_1}{x-r_1} + \dots + \frac{a_n}{x-r_n}$$

(In other words, prove that this partial fraction decomposition that you learned in calculus always exists and is unique.)