Math 223, Section 101—Homework #3

due Wednesday, September 29, 2010 at the beginning of class

Practice problems. Don't turn in these problems. Most of these problems are easy, and most of them have answers in the back of the book, so they can be useful to you to check your understanding of the material.

1. Friedberg, Insel, and Spence, Section 1.2, p. 16, #21
2. Friedberg, Insel, and Spence, Section 1.5, p. 42, #18
3. In the vector space $P(\mathbb{R})$, consider the subset $S = \{1-x, x-x^2, x^2-x^3, \dots\}$. Notice the telescoping infinite series
$$1 = (1-x) + (x-x^2) + (x^2-x^3) + \cdots .$$
Despite this, show that 1 is *not* in $\operatorname{span}(S)$.
4. Friedberg, Insel, and Spence, Section 1.6, pp. 53–54, #1(a)–(l)
5. Friedberg, Insel, and Spence, Section 1.6, p. 54, #2(a),(c),(e), #3(a),(c),(e), #4, #5, and #7
6. Friedberg, Insel, and Spence, Section 1.6, p. 55, #9, #10(a),(c), and #13
7. Friedberg, Insel, and Spence, Section 1.6, p. 56, #19 and #24
8. Friedberg, Insel, and Spence, Section 1.6, p. 57, #26

Homework problems. Write up and hand in solutions to these problems. Please have your solutions to these problems in the correct order on your pages. Write clearly and legibly, in complete sentences; your solutions will be better if you think about how to best phrase your answer before you begin to write it. Please staple the pages together (unstapled solutions will not be accepted).

1. Friedberg, Insel, and Spence, Section 1.5, p. 42, #13. Ignore the bit about "a field of characteristic not equal to two": let V be simply a regular vector space with real numbers as its scalars.
2. Friedberg, Insel, and Spence, Section 1.6, p. 54, #2(b),(d) and #3(b),(d)
3. Friedberg, Insel, and Spence, Section 1.6, p. 55, #10(b),(d)
4. Friedberg, Insel, and Spence, Section 1.6, p. 55, #14, except use $\mathbb{R}^5$ rather than the book's F^5.
5. Friedberg, Insel, and Spence, Section 1.6, p. 56, #15, except use $M_{n\times n}(\mathbb{R})$ rather than the book's $M_{n\times n}(F)$.
6. Friedberg, Insel, and Spence, Section 1.6, p. 56, #22 and #25

(continued on next page)

7. Let V be the set of all infinite sequences of real numbers; a typical element of V looks like $\{a_1, a_2, a_3, \dots\}$. With addition and scalar multiplication defined componentwise, V is a vector space (you don't have to prove this). For each positive integer j, define e_j to be the element of V with a 1 in the jth position and 0s everywhere else:
$$e_j = \{\underbrace{0, \dots, 0}_{j-1 \text{ 0s}}, 1, 0, 0, \dots\}.$$
Let $E = \{e_j \colon j \geq 1\}$ be the set of all these vectors e_j.
 (a) Prove that E is linearly independent.
 (b) Prove that E is *not* a basis for V.

8. Let V be a vector space, let $x_1, \dots, x_n$ be vectors in V, and let $W = \operatorname{span}\{x_1, \dots, x_n\}$. Define
$$B = \big\{x_j \colon x_j \notin \operatorname{span}(\{x_1, \dots, x_{j-1}\})\big\};$$
that is, B is the set of all those x_j (where j ranges from 1 to n) that are *not* in the span of the x_i's that come before it. Prove that B is a basis for W. (To decide what to do with x_1: we interpret $\{x_1, \dots, x_{j-1}\}$ as the empty set when $j = 1$; recall that $\operatorname{span}(\emptyset) = \{\underline{0}\}$.) Do not use arguments involving reduced row echelon forms—our knowledge of that was derived *from* the result in this problem.

Bonus question. Find, with proof, a vector space other than $P(\mathbb{R})$ that has a countably infinite basis. (If you haven't seen it before, for a discussion of countability see

`http://en.wikipedia.org/wiki/Countable`

or most textbooks that discuss proofs and sets.)