Math 223, Section 101—Homework #6

due Friday, October 29, 2010 at the beginning of class

Practice problems. Don't turn in these problems. Most of these problems are easy, and most of them have answers in the back of the book, so they can be useful to you to check your understanding of the material.

- 1. Friedberg, Insel, and Spence, Section 3.2, pp. 165–6, #1, #2(a),(c),(e),(g), #4(a), and #5(a),(c),(e),(g)
- 2. Friedberg, Insel, and Spence, Section 3.2, pp. 166–7, #6(a),(c),(e) and #7
- 3. Friedberg, Insel, and Spence, Section 3.2, p. 168, #20(a)
- 4. Friedberg, Insel, and Spence, Section 3.3, p. 179, #1. In (h), replace F^n by \mathbb{R}^n .
- 5. Friedberg, Insel, and Spence, Section 3.3, pp. 179–180, #2(a),(c),(e),(g) and #3(a),(c),(e),(g)
- 6. Friedberg, Insel, and Spence, Section 3.3, pp. 180-1, #4, #6, and #7
- 7. Friedberg, Insel, and Spence, Section 4.4, p. 236, #1 and #2(a). In #1(h),(i), replace $M_{n \times n}(F)$ by $M_{n \times n}(\mathbb{R})$.
- 8. Friedberg, Insel, and Spence, Section 4.4, pp. 236–7, #3(a),(c),(g) and 4(a),(c),(g)
- 9. Let V and W be finite-dimensional vector spaces. Let

 $T_1: V \to V, \qquad T_2: V \to W, \qquad \text{and } T_3: W \to W$

be linear transformations. If T_1 and T_3 are invertible, prove that $rank(T_2T_1) = rank(T_2) = rank(T_2T_3)$.

Homework problems. Write up and hand in solutions to these problems. Please have your solutions to these problems in the correct order on your pages. Write clearly and legibly, in complete sentences; your solutions will be better if you think about how to best phrase your answer before you begin to write it. Please staple the pages together (unstapled solutions will not be accepted).

- 1. Friedberg, Insel, and Spence, Section 3.2, p. 166, #5(b),(d),(f),(h)
- 2. Friedberg, Insel, and Spence, Section 3.2, pp. 166-7, #6(b),(d),(f)
- 3. Friedberg, Insel, and Spence, Section 3.2, p. 168, #14
- 4. Friedberg, Insel, and Spence, Section 3.3, pp. 179–180, #2(b),(d) and #3(b),(d). Hint: from the numbers 0, 1, 1, 1, 1, 2 you can assemble two column vectors, one of which is a particular solution to the system in #3(b) and the other of which is a particular solution to the system in #3(d).
- 5. Friedberg, Insel, and Spence, Section 3.3, p. 181, #10
- 6. Friedberg, Insel, and Spence, Section 4.4, pp. 236–7, #3(b),(d),(h); also, calculate the determinant of the 4 × 4 matrix in problem #4(h) by using the forward pass of Gaussian elimination to transform the matrix into row echelon form.

- 7. (a) Write down the definition of two matrices being *similar*. (One-word hint: index.) Suppose that the two matrices A and B are similar. Prove that det(A) = det(B).
 - (b) Let A be an $m \times n$ matrix with $m \neq n$. Suppose that there exists an $n \times m$ matrix B such that $BA = I_n$. Prove that there does not exist an $n \times m$ matrix C such that $AC = I_m$. (One-word hint: rank.)
- 8. Friedberg, Insel, and Spence, Section 4.4, p. 238, #6, except replace $M_{n \times n}(F)$ by $M_{n \times n}(\mathbb{R})$

Bonus question. Friedberg, Insel, and Spence, Section 3.2, p. 168, #18