

## Math 223, Section 101—Homework #7

due Friday, November 5, 2010 at the beginning of class

**Practice problems.** Don't turn in these problems. Most of these problems are easy, and most of them have answers in the back of the book, so they can be useful to you to check your understanding of the material.

1. I have linked to a web page with practice problems about complex numbers. You can go to the URL <http://tinyurl.com/MATH223> or follow the link from the course web page. I'm sure there are lots of similar practice problems out there.
2. Friedberg, Insel, and Spence, Section 4.2, pp. 220–1, #1 and #3. In 1(a),(g), change  $M_{n \times n}(F)$  and  $F$  to  $M_{n \times n}(\mathbb{R})$  and  $\mathbb{R}$ .
3. Friedberg, Insel, and Spence, Section 4.3, p. 228, #1(a)–(f). In (b)–(e), change  $M_{n \times n}(F)$  to  $M_{n \times n}(\mathbb{R})$ .
4. Friedberg, Insel, and Spence, Section 4.3, p. 229, #11 and #12
5. Friedberg, Insel, and Spence, Section 4.3, p. 231, #26(a),(c),(e),(g)
6. Friedberg, Insel, and Spence, Section 4.4, pp. 236–7, #2(c), #3(e), and #4(e)
7. Friedberg, Insel, and Spence, Section 5.1, p. 256, #1(a)–(f),(h)–(k). In (h), change  $F$  and  $F^n$  to  $\mathbb{R}$  and  $\mathbb{R}^n$ .
8. Friedberg, Insel, and Spence, Section 5.1, pp. 256–7, #2(a),(c),(e) and #3(a),(c)
9. Friedberg, Insel, and Spence, Section 5.1, pp. 257–8, #4(a),(b),(f),(h)–(j) and #5

**Homework problems.** Write up and hand in solutions to these problems. Please have your solutions to these problems in the correct order on your pages. Write clearly and legibly, in complete sentences; your solutions will be better if you think about how to best phrase your answer before you begin to write it. Please staple the pages together (unstapled solutions will not be accepted).

1. Friedberg, Insel, and Spence, Section 4.4, pp. 236–7, #3(f) and #4(f)
2. (a) Solve the following system of linear equations over  $\mathbb{C}$ :

$$\begin{array}{rrcr} z_1 + & (-2)z_2 + & (-5i)z_3 = & -2 \\ -2z_1 + & (3 + 3i)z_2 + & (-7 + 11i)z_3 = & 2i \\ -iz_1 + & & z_2 + & = 3 + i \end{array}$$

That is, find all ordered triples  $(z_1, z_2, z_3)$  of complex numbers such that the above equations are satisfied. (It's easy to make mistakes in the calculations: check your answer!)

(b) What is  $\det \begin{pmatrix} 1 & -2 & -5i \\ -2 & 3 + 3i & -7 + 11i \\ -i & 1 & 0 \end{pmatrix}$ ? Why? (This should be a one-liner.)

3. Friedberg, Insel, and Spence, Section 5.1, pp. 257–8, #3(d) and #4(g)
4. Friedberg, Insel, and Spence, Section 4.3, p. 230, #24. The answer is in the back of the book, but you will need to justify why it is the right answer.

(continued on next page)

5. Let  $A$  and  $B$  be invertible  $n \times n$  matrices. Let  $\alpha$  be the subset of  $\mathbb{R}^n$  consisting of the  $n$  columns of  $A$  (ordered from left to right), and let  $\beta$  be the subset of  $\mathbb{R}^n$  consisting of the  $n$  columns of  $B$  (ordered from left to right). Prove that  $\alpha$  and  $\beta$  are ordered bases for  $\mathbb{R}^n$ , and prove that the change of coordinates matrix from  $\beta$  to  $\alpha$  is  $A^{-1}B$ .
6. Call an  $n \times n$  matrix “backwards-upper-triangular” if all of its entries below the northeast-to-southwest diagonal equal 0; call it “backwards-lower-triangular” if all of its entries above the northeast-to-southwest diagonal equal 0. For example, when  $n = 3$ , these matrices look like

$$\begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix},$$

respectively, where each  $*$  denotes any real number. Suppose that  $A$  is a backwards-upper-triangular matrix such that all of the entries on the northeast-to-southwest diagonal are nonzero. Prove that  $A$  is invertible and that  $A^{-1}$  is a backwards-lower-triangular matrix.

7. Friedberg, Insel, and Spence, Section 5.1, pp. 258–9, #11
8. (a) Define  $T : P_3(\mathbb{R}) \rightarrow P_3(\mathbb{R})$  by  $T(p(x)) = x^3p(-1/x)$ . Prove that  $T$  does not have any eigenvectors. (Hint: find the characteristic polynomial of  $T$  with respect to the standard ordered basis for  $P_3(\mathbb{R})$ .)
- (b) Define  $T : P_3(\mathbb{C}) \rightarrow P_3(\mathbb{C})$  by  $T(p(x)) = x^3p(-1/x)$ . Find an eigenvector for  $T$ . ( $P_3(\mathbb{C})$  is the set of polynomials of degree at most 3, where complex numbers are allowed as the coefficients.)

**Bonus question.** Friedberg, Insel, and Spence, Section 5.1, p. 259, #12