Math 223, Section 101—Homework #8
due Friday, November 19, 2010 at the beginning of class

Practice problems. Don't turn in these problems. Most of these problems are easy, and most of them have answers in the back of the book, so they can be useful to you to check your understanding of the material.

1. Friedberg, Insel, and Spence, Section 5.2, p. 279, #1(a)–(g). Hint for (c): the zero vector.
2. Friedberg, Insel, and Spence, Section 5.2, pp. 279–280, #2(a),(c),(e),(g) and #3(a),(c)–(e)
3. Friedberg, Insel, and Spence, Section 5.2, p. 280, #7. Hint: Example 7.
4. Friedberg, Insel, and Spence, Section 5.4, p. 321, #1(e),(f)
5. Friedberg, Insel, and Spence, Section 5.4, p. 323, #10(a),(c), but only the "find the characteristic polynomial $f(t)$ of T" part.
6. Friedberg, Insel, and Spence, Section 5.4, p. 324, #19
7. Friedberg, Insel, and Spence, Section 6.1, p. 336, #1(a),(c)–(h), #2, and #3
8. Friedberg, Insel, and Spence, Section 6.1, p. 338, #16(b) and #17

Homework problems. Write up and hand in solutions to these problems. Please have your solutions to these problems in the correct order on your pages. Write clearly and legibly, in complete sentences; your solutions will be better if you think about how to best phrase your answer before you begin to write it. Please staple the pages together (unstapled solutions will not be accepted).

1. Friedberg, Insel, and Spence, Section 5.2, pp. 279–280, #2(d),(f) and #3(f)
2. Friedberg, Insel, and Spence, Section 5.2, p. 282, #17 and #18. These problems look intimidating, but each part has an extremely short proof.
3. Let V be a finite-dimensional vector space (over $\mathbb{R}$) whose dimension is odd. Prove that every linear operator on V has an eigenvector.
4. Friedberg, Insel, and Spence, Section 5.4, p. 323, #18(a),(b)
5. Friedberg, Insel, and Spence, Section 6.1, p. 337, #8
6. Friedberg, Insel, and Spence, Section 6.1, p. 337, #12
7. Let A be an $n \times n$ matrix, so that $A, A^2, A^3, \dots$ are all elements of $M_{n\times n}(\mathbb{R})$. Prove that
$$\dim\big(\operatorname{span}\{A, A^2, A^3, \dots\}\big) \le n.$$
8. Friedberg, Insel, and Spence, Section 6.1, p. 338, #18

Bonus question. Suppose that $A \in M_{4\times 4}(\mathbb{R})$ satisfies $A^3 = -A$. Determine all possibilities for the characteristic polynomial of A. For each possible characteristic polynomial, determine all possibilities for $\operatorname{rank}(A)$. Give examples to show that your possibilities really are possible. (Hint: there are three possible characteristic polynomials. One approach to this problem involves the Cayley–Hamilton Theorem.)