Math 223, Section 201
Homework #2
due Friday, January 25, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Lay, Section 1.4, p. 46, #20

II. Lay, Section 1.4, p. 47, #33

III. Lay, Section 1.5, p. 56, #8 (Note: the matrix is a *coefficient* matrix)

IV. If the statement is true, prove it (do not simply justify it intuitively); if the statement is false, give a counterexample.
   (a) Three vectors in $\mathbb{R}^4$ can never span all of $\mathbb{R}^4$.
   (b) Four vectors in $\mathbb{R}^3$ can never span all of $\mathbb{R}^3$.
   (c) If A is a 3×4 matrix whose columns do not span $\mathbb{R}^3$, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some vector $\mathbf{b}$ in $\mathbb{R}^3$.

V. Lay, Section 1.5, p. 57, #29–32

January 25's quiz will be one of these five questions.

Homework Questions—hand these in

I. Let $A = [\mathbf{a}_1\, \mathbf{a}_2\, \mathbf{a}_3]$, where

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -4 \\ 7 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} -2 \\ 5 \\ -8 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 3 \\ -6 \\ 9 \end{bmatrix}.$$

Find the set of solutions to $A\mathbf{x} = \mathbf{0}$ (write it in parametric vector form). Use this to write down the set of solutions to each of the systems $A\mathbf{x} = \mathbf{a}_1$, $A\mathbf{x} = \mathbf{a}_2$ and $A\mathbf{x} = \mathbf{a}_3$.

II. Lay, Section 1.5, p. 57, #26 and #28

III. Let A be a 3×3 matrix.
   (a) Suppose that the entries in each row of A add to zero. If $\mathbf{b}$ is in $\mathbb{R}^3$, prove that the equation $A\mathbf{x} = \mathbf{b}$ either is inconsistent or else has infinitely many solutions.
   (b) Suppose that the entries in each column of A add to zero. If $\mathbf{b}$ is in $\mathbb{R}^3$, prove that the equation $A\mathbf{x} = \mathbf{b}$ either is inconsistent or else has infinitely many solutions.

(continued on back of page)

IV. (This problem is concerned with balancing chemical reactions. For a description of what this means together with a worked example, see Lay, Section 1.5, p. 57, #39 and the corresponding solution in the back of the book.)

(a) Balance the chemical reaction representing the combustion of propane (with carbon dioxide and water as the products):

$$C_3H_8 + O_2 \rightarrow CO_2 + H_2O$$

There will be infinitely many solutions to the mathematical problem; single out the one that makes the most sense chemically.

(b) Same question for the combustion of octane:

$$C_8H_{18} + O_2 \rightarrow CO_2 + H_2O$$

(c) Now consider the simultaneous combustion of propane and octane:

$$C_3H_8 + C_8H_{18} + O_2 \rightarrow CO_2 + H_2O$$

Find all of the solutions to the mathematical problem of balancing this reaction. How does the answer to this part of the problem relate to the answers of the first two parts?