Math 223, Section 201
Homework #3

due Friday, February 1, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Lay, Section 1.6, p. 65, #7

II. Lay, Section 1.6, p. 66, #9 and #31–36

III. For each of the following sets of vectors, either: find an additional vector that you can add to the set so that the new, bigger set is still linearly independent (justifying why the new set is linearly independent); or else, explain why this is impossible.

(a) $\left\{\begin{bmatrix} 2 \\ -3 \end{bmatrix}\right\}$ (b) $\left\{\begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \end{bmatrix}\right\}$

(c) $\left\{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}\right\}$ (d) $\left\{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}\right\}$

IV. Lay, Section 1.7, p. 74, #6–8

V. Lay, Section 1.7, p. 75, #30–31

February 1's quiz will be a question like one of these questions.

Homework Questions—hand these in

I. Lay, Section 1.6, p. 66, #40

II. Let $\{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_6\}$ be a set of vectors in $\mathbb{R}^5$. Suppose that the pivot columns of the matrix $[\mathbf{a}_1\, \mathbf{a}_2\, \ldots\, \mathbf{a}_6]$ are the first, third, and fourth columns.

(a) Prove that the set $\{\mathbf{a}_1, \mathbf{a}_3, \mathbf{a}_4\}$ is linearly independent.
(b) Prove that the set $\{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_6\}$ is linearly dependent.
(c) Prove that $\text{Span}\{\mathbf{a}_1, \mathbf{a}_3, \mathbf{a}_4\} = \text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_6\}$.

(This is an example of the general fact I tentatively mentioned in class: the vectors corresponding to the pivot columns form a linearly independent set, and somehow throwing away the remaining vectors "removes the redundancy" in terms of all possible linear combinations.)

III. Lay, Section 1.7, p. 74, #10 and #12

IV. Find a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^4$ whose range contains each of the vectors $\begin{bmatrix} 3 \\ 1 \\ -1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 4 \\ 0 \\ 1 \\ -2 \end{bmatrix}$, and $\begin{bmatrix} 5 \\ 3 \\ -4 \\ 2 \end{bmatrix}$. (Something to ponder: why is this a *harder* question with $\mathbb{R}^2$ as the domain than it would be with $\mathbb{R}^3$ as the domain?)