Math 223, Section 201
Homework #5
due Friday, March 1, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Calculate the inverse of each of the following matrices, if the inverse exists.

(a) $\begin{bmatrix} 4 & 11 \\ 18 & 25 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 10 \\ 18 & 45 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 & 0 \\ 4 & 1 & 1 \\ 3 & -3 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 & 0 \\ 4 & 1 & 1 \\ 3 & -2 & 1 \end{bmatrix}$

March 1's quiz will a question like this warm-up question (but not as long).

Homework Questions—hand these in

I. Lay, Section 2.7, p. 153, #4

II. Let $A = \begin{bmatrix} -1 & 0 & -3 \\ -2 & 1 & 3 \\ 1 & -3 & -2 \end{bmatrix}$. Find a solution $\mathbf{x} \neq \mathbf{0}$ to the equation $A\mathbf{x} = -2\mathbf{x}$.

III. A cubic polynomial has the form $p(x) = c_0 + c_1x + c_2x^2 + c_3x^3$, where the coefficients c_0, c_1, c_2, c_3 are real numbers.

(a) Suppose you know that the cubic polynomial $p(x)$ satisfies $p(1) = 3$, $p(2) = 4$, and $p(3) = 1$. Write down a system of linear equations that the coefficients c_0, c_1, c_2, c_3 must satisfy.

(b) Solve the system you wrote down in part (a), writing your answer in vector parametric form.

(c) Find all cubic polynomials $q(x)$ such that $q(1) = q(2) = q(3) = 0$. You should be able to do this without any work, given your answer to part (b).

IV. Let $a, b, c, d, e, f, g, h, i$ be real numbers. Suppose that the set $\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \begin{bmatrix} d \\ e \\ f \end{bmatrix}, \begin{bmatrix} g \\ h \\ i \end{bmatrix} \right\}$ is linearly independent. Prove that the set $\left\{ \begin{bmatrix} a \\ d \\ g \end{bmatrix}, \begin{bmatrix} b \\ e \\ h \end{bmatrix}, \begin{bmatrix} c \\ f \\ i \end{bmatrix} \right\}$ spans $\mathbb{R}^3$. (Note that these two sets are not the same!)

(continued on back of page)

V. (a) Write down a specific linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ that is onto but not one-to-one. (You can choose whatever m and n you like. You don't have to justify why your T is onto and not one-to-one, but be sure that you're right!)

(b) Given the transformation T you wrote down in part (a), are there vectors $\mathbf{b} \in \mathbb{R}^m$ such that the equation $T(\mathbf{x}) = \mathbf{b}$ has no solution? exactly one solution? infinitely many solutions? For each of these three questions, either give a specific example of such a vector $\mathbf{b}$ or else explain why no such vector exists.

(c) Write down a specific linear transformation $U : \mathbb{R}^q \to \mathbb{R}^p$ that is one-to-one but not onto. (You can choose whatever p and q you like. You don't have to justify why your U is one-to-one and not onto, but be sure that you're right!)

(d) Given the transformation U you wrote down in part (c), are there vectors $\mathbf{b} \in \mathbb{R}^p$ such that the equation $U(\mathbf{x}) = \mathbf{b}$ has no solution? exactly one solution? infinitely many solutions? For each of these three questions, either give a specific example of such a vector $\mathbf{b}$ or else explain why no such vector exists.

VI. Let A be a 3×5 matrix and B a 5×3 matrix such that AB is the zero matrix. Let $C = BA$. Suppose that $\mathbf{b}$ is a vector in $\mathbb{R}^5$ such that the equation $C\mathbf{x} = \mathbf{b}$ is consistent. Prove that $\mathbf{b}$ is a solution to the homogeneous equation $C\mathbf{x} = \mathbf{0}$.