Math 223, Section 201
Homework #6

due Friday, March 8, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Lay, Section 3.2, p. 194, #29

II. Lay, Section 3.2, p. 194, #32

III. Lay, Section 3.3, p. 204, #9

IV. Lay, Section 3.3, p. 204, #18

V. Lay, Section 3.3, p. 205, #29 and #30

VI. Lay, Chapter 3 Supplementary Exercises, p. 206, #1

March 8's quiz will be one of the first five warm-up questions.

Homework Questions—hand these in

I. Lay, Section 3.2, p. 194, #22 and #26

II. Lay, Section 3.2, p. 194, #34

III. Lay, Section 3.3, p. 204, #6

IV. Let A be an $m \times n$ matrix and B an $n \times m$ matrix, so that AB is $m \times m$. Suppose that $\det AB = 7$. Prove that the columns of A span $\mathbb{R}^m$ and that the columns of B are linearly independent. (Note: A and B are not necessarily square matrices!)

V. Lay, Section 3.3, p. 204–5, #22 and #24

VI. Given real numbers $x_1, x_2, \dots, x_n$, define the *Vandermonde matrix* $V(x_1, x_2, \dots, x_n)$ to be the $n \times n$ matrix

$$\begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \dots & x_2^{n-1} \\ \vdots & & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^{n-1} \end{bmatrix}.$$

Prove that

$$\begin{aligned} \det V(x_1, x_2, \dots, x_n) &= (x_2 - x_1)(x_3 - x_1)\dots(x_n - x_1) \\ &\quad \times (x_3 - x_2)(x_4 - x_2)\dots(x_n - x_2) \times \dots \times (x_n - x_{n-1}) \\ &= \prod_{1 \le i < j \le n} (x_j - x_i). \end{aligned}$$

Conclude that $V(x_1, x_2, \dots, x_n)$ is invertible if and only if the numbers $x_1, x_2, \dots, x_n$ are all distinct.