Math 223, Section 201
Homework #7
due Friday, March 15, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Lay, Section 4.3, p. 238–239, #23, #24, #25, #29, and #30

March 15's quiz will be one of these five warm-up questions.

Homework Questions—hand these in

I. Let V be the set of all points on the circle of radius 1 centered at the origin in $\mathbb{R}^2$. Every point on the circle (x, y) is represented by an angle θ, by the formula $(x, y) = (\cos\theta, \sin\theta)$; for example, $(0, -1)$ is represented by $270°$, since $(\cos 270°, \sin 270°) = (0, -1)$. Notice that each point in V is represented by more than one angle θ; for instance, $(0, -1)$ is also represented by $-90°$.

Define addition and scalar multiplication on V by the following rules: if two points $\mathbf{x}$ and $\mathbf{y}$ in V are represented by the angles θ_1 and θ_2, then their sum $\mathbf{x} + \mathbf{y}$ is the point in V represented by the angle $\theta_1 + \theta_2$. If $r \in \mathbb{R}$, and $\mathbf{x} \in V$ is represented by the angle θ with $0° \le \theta < 360°$, then $r\mathbf{x}$ is the point in V represented by the angle $r\theta$.

Of the ten properties of vector spaces listed in the definition on page 211 of Lay, which one(s) do V satisfy and which one(s) don't V satisfy? Give brief justifications for your answers. Is V a vector space?

II. Let $A = \begin{bmatrix} 1 & 5 & 6 & 2 & 7 \\ 1 & 3 & 4 & 2 & 5 \\ 2 & 7 & 9 & 6 & 11 \\ 0 & 2 & 2 & -1 & 2 \end{bmatrix}$. Find a basis for each of for $\operatorname{Col} A$, $\operatorname{Nul} A$, and $\operatorname{Row} A$.

III. Recall that $D\colon \mathbb{P} \to \mathbb{P}$ denotes the differentiation linear transformation. Find a specific linear transformation $T\colon \mathbb{P} \to \mathbb{P}$ such that $D \circ T$ is the identity transformation on $\mathbb{P}$, i.e., such that $D(T(\mathbf{x})) = \mathbf{x}$ for all $\mathbf{x} \in \mathbb{P}$. What is $T(D(\mathbf{x}))$?

IV. Write down a linear transformation $T\colon \mathbb{R}^2 \to C$, where C denotes the vector space of circles $\{C(r)\colon r > 0\}$ from our example in class, with addition and scalar multiplication defined by $C(r) + C(s) = C(rs)$ and $kC(r) = C(r^k)$. Your linear transformation must not send everything in $\mathbb{R}^2$ to the same vector. Calculate the kernel and range of your linear transformation.

V. Let $\mathbb{S}$ be the vector space of doubly infinite sequences (see Example 3 in Section 4.1 of Lay). A sequence $\{y_k\} \in \mathbb{S}$ is *periodic* if there exists a positive integer p (the *period*) such that $y_{k+p} = y_k$ for every integer k. For example, the sequence $(\dots, 2, 5, 9, 2, 5, 9, 2, 5, 9, \dots)$ is periodic with period $p = 3$; the sequence $\left\{\cos\left(\frac{k\pi}{4}\right)\right\} = (\dots, 1, \frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2}, -1, -\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 1, \dots)$ is periodic with period $p = 8$; and any constant sequence is periodic with period $p = 1$.

Let H be the set of all periodic sequences in $\mathbb{S}$. Is H a subspace of $\mathbb{S}$? Prove your answer.