Math 223, Section 201
Homework #8

due Monday, March 25, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Lay, Section 4.4, p. 248–249, #4 and #8

II. Lay, Section 4.4, p. 248–249, #6 and #28

III. Lay, Section 4.5, p. 255, #12

IV. Let $T : V \to W$ be a linear transformation between two vector spaces V, W. Let $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ be a set of vectors in V, and let $\mathbf{w}_1 = T(\mathbf{v}_1)$, …, $\mathbf{w}_p = T(\mathbf{v}_p)$.
   (a) If $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ is linearly dependent, prove that $\{\mathbf{w}_1, \dots, \mathbf{w}_p\}$ is also linearly dependent.
   (b) Give an example to show that it is possible for $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ to be linearly independent while $\{\mathbf{w}_1, \dots, \mathbf{w}_p\}$ is linearly dependent.

V. Define $H = \{p(t) \in \mathbb{P}_4 \colon p(0) = p(1) = p(2)\}$. Prove that H is a subspace of $\mathbb{P}_4$.

March 25's quiz will be one of these five warm-up questions.

Homework Questions—hand these in

I. Lay, Section 4.4, p. 248–249, #14 and #30

II. Let $T : V \to W$ be a linear transformation between two vector spaces V, W. Let $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ be a set of vectors in V, and let $\mathbf{w}_1 = T(\mathbf{v}_1)$, …, $\mathbf{w}_p = T(\mathbf{v}_p)$.
   (a) Give an example to show that it is possible for $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ to span V while $\{\mathbf{w}_1, \dots, \mathbf{w}_p\}$ does not span W.
   (b) Suppose T is onto. Prove that if $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ spans V, then $\{\mathbf{w}_1, \dots, \mathbf{w}_p\}$ spans W.

III. Lay, Section 4.5, p. 255, #8

IV. Let $B = \{\mathbf{b}_1, \dots, \mathbf{b}_n\}$ be a basis for $\mathbb{R}^n$, and let $\mathbf{y}_1, \dots, \mathbf{y}_n$ be any vectors in $\mathbb{R}^n$. Show that there is a unique linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ such that $T(\mathbf{b}_1) = \mathbf{y}_1$, …, $T(\mathbf{b}_n) = \mathbf{y}_n$. (Hint: try to relate the standard matrix of the linear transformation to the change-of-coordinates matrix P_B.)

V. Let H be the subspace of $\mathbb{P}_4$ defined in Warm-Up Question V.
   (a) Find a basis for H. (Hint: use the coordinate mapping relative to the standard basis for $\mathbb{P}_4$ so you can think about the problem in the appropriate $\mathbb{R}^n$.)
   (b) Let $q(t) = t^4 - 2t^3 - t^2 + 2t + 1$. Note that $q(0) = q(1) = q(2) = 1$, so that $q \in H$. Find a basis for H that has $q(t)$ as one of its vectors.

VI. Let A be an $m \times n$ matrix whose rank equals r. Show that there exists an $r \times r$ minor of A that is invertible. Also show that if k is an integer satisfying $k \le m$, $k \le n$, and $k > r$, then every $k \times k$ minor of A is singular.