Math 308 Homework #1 due Wednesday, Septemeber 15, 2004 at 11 AM

When writing your solutions for problems in this course, keep the following ideas in mind: your solutions should be written in complete sentences and understandable paragraphs (not "two-column proofs"). State what axioms, lemmas, or theorems you are using when you use them, either in words or by their name (or number from Baragar's book). In short, your proofs should take the same form as the proofs in the textbook.

On all your assignments, you may use anything mentioned in class (with the obvious exception of the statement of the problem you're trying to solve, if it was brought up) and anything from previous homeworks in the course. You may also use earlier problems from the same homework to solve later problems on that homework, regardless of whether you correctly solved the earlier problem. For instance, you could use the statement of problem II to solve problem IV, if it was helpful.

For this assignment, you may use anything in the textbook up to and including page 16.

- I. Prove that the composition of two isometries is an isometry. In other words, suppose that the two functions f and g from the plane to itself are both isometries, and define another function h from the plane to itself by h(P) = f(g(P)); prove that h is an isometry.
- II. Suppose that the isometry f fixes two distinct points A and B. Prove that it fixes every point C on the line AB. (Note: to say that f fixes a point P is simply to say that f(P) = P.) (Hint: use the statement of Exercise 1.11, which we mentioned in class. Split into cases depending on which of the three points A, B, C is between the other two.)
- III. Given a line ℓ and a point *C* that is not on ℓ , let *f* be the isometry that fixes every point on ℓ and no other points, and let C' = f(C). Prove that ℓ is the perpendicular bisector of the segment *CC'*. Conclude that there exists a point *D* on ℓ such that the segment *CD* is perpendicular to ℓ .
- IV. Prove the "SAS Theorem": if $\triangle ABC$ and $\triangle A'B'C'$ are triangles with |AB| = |A'B'|and |BC| = |B'C'| and with angle *ABC* congruent to angle A'B'C', then $\triangle ABC \equiv \triangle A'B'C'$. (Hint: the fact that two angles are congruent means that there exists an isometry with certain properties—start there.)