## Math 308 Homework #5

## due Wednesday, October 27, 2004 at 11 AM

For this assignment, *unless otherwise noted*, you may use anything in Chapters 1 and 2 and Sections 3.1–3.5 of the textbook, *including* the assertions stated in the exercises. (You may always use things mentioned in class, problems from previous homeworks, and earlier problems from the same homework.) You may also use the facts—lemmas, theorems, corollaries—proved in Section 3.6, but *not* the exercises from those sections (other than the ones mentioned in class, etc.), unless you prove them yourself.

- I. Using Theorem 3.6.5, prove that there is no construction that "pentasects" an arbitrary angle. That is, show that there is a constructible angle measuring  $x^{\circ}$  such that it is impossible to construct an angle measuring  $(x/5)^{\circ}$ .
- II. Let  $C_A(|AB|)$  be a circle centered at the constructed point *A* and going through the constructed point *B*, and let *D* be any constructed point outside the circle *C*. Describe, with proof, a construction of a line going through *D* that is tangent to the circle *C*.
- III. Given three distinct, already constructed points *A*, *B*, and *C*, demonstrate that the Euler line of the triangle  $\triangle ABC$  can be constructed.
- IV. Prove the formula

$$4 \arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right) = 45^{\circ}.$$

Using this formula, describe a construction of a segment of length 1/239 units that does *not* use Lemmas 3.4.2 or 3.4.3 (in other words, one that does *not* rely on multiplication or inversion of already constructed lengths).

Hint: remind yourself of the formulas for  $\tan(\alpha + \beta)$  and  $\tan(2\alpha)$  in terms of  $\tan \alpha$  and  $\tan \beta$ . If you let  $x^{\circ}$  and  $y^{\circ}$  be the angles such that  $\tan x^{\circ} = \frac{1}{5}$  and  $\tan y^{\circ} = \frac{1}{239}$ , then reduce the problem to showing that  $\tan(2(2x^{\circ})) = \tan(y^{\circ} + 45^{\circ})$ .

V. Let  $d = 2 \cos 40^\circ$ . Based on the fact that  $3 \cdot 40^\circ = 120^\circ$  and that  $\cos 120^\circ$  is a known quantity, find (with proof) an polynomial f(x) with integer coefficients, irreducible over the integers, such that f(d) = 0. Conclude, *without* using Theorem 3.6.5, that it is impossible to construct a regular nonagon (nine-sided polygon).