## Math 308 Homework #6

due Friday, November 5, 2004 at 11 AM

For this assignment, you may use anything in Chapters 1–3, *including* the assertions stated in the exercises. (You may always use things mentioned in class, problems from previous homeworks, and earlier problems from the same homework.) You may also use the facts—lemmas, theorems, corollaries—proved in Sections 6.1–6.4, but *not* the exercises from those sections (other than the ones mentioned in class, etc.), unless you prove them yourself.

All problems refer to *hyperbolic* lines and angles unless otherwise stated. For the problems that say to "draw" diagrams in the disk model of the hyperbolic plane, drawing them freehand is fine as long as they are reasonably accurate.

I. List all of the numbers *n* between 2,000 and 3,000 for which the regular *n*-gon can be constructed. Describe briefly why your list is correct.

Recall that two integers *m* and *n* are *relatively prime* if gcd(m, n) = 1.

- II. Prove Theorem 3.6.3. You may use the following number theory (Math 312) fact without having to prove it: if *m* and *n* are relatively prime, then there exist positive integers *x* and *y* such that mx ny = 1.
- III. Let  $\angle PQA$  be a right angle, and let  $\varepsilon$  be any positive number. Show that there exists a point *R* on the ray *QA* such that  $\angle PRQ < \varepsilon^{\circ}$ . (Other than using a point *A* on the ray instead of a "point *M* at infinity", this is the same problem as Baragar, p. 125, #6.7. The diagram there provides a good hint.)
- IV. Baragar, p. 125, #6.8
- V. Suppose you have a Euclidean circle centered at *O* with two perpendicular diameters *MN* and *AB*. Let *P* be a point on the segment *AB* other than *O*, and let  $\alpha^{\circ}$  be the measure of the (Euclidean) angle  $\angle PMN$ . Now think of the circle as forming the boundary of the hyperbolic plane, so that the diameter *MN* is a hyperbolic-line  $\ell$  and *O* and *P* are points in the hyperbolic plane. Prove that the angle of parallelism  $\Pi(|PO|)$  has the value  $(90 2\alpha)^{\circ}$ . (Hint: once you draw the hyperbolic-lines through *P* that are parallel to  $\ell$ , think about the whole drawing as Euclidean again.)
- VI. Using the disk model of the hyperbolic plane, draw a triangle  $\triangle ABC$  with three  $45^{\circ}$  angles, where *A* is the Euclidean-center of the Euclidean-circle that forms the boundary of the hyperbolic plane. If *O* is the Euclidean-center of the Euclidean-circle that defines the line *BC*, prove that *ABOC* is a Euclidean-rhombus. Now consider the isometry *f* that is the reflection of the hyperbolic plane in the line *BC*. Draw the images of the lines *AB* and *AC* under *f* and their intersection *A'*. What are the measures of the angles  $\angle ABA'$  and  $\angle ACA'$ ? Conclude that the Euclidean-center of the Euclidean-circle defining the line *A'B* lies on the Euclidean-line *AB*.