Math 308 Homework #7 due Monday, November 22, 2004 at 11 AM

For this assignment, *unless otherwise noted*, you may use anything in Chapters 1–3 and 6, *including* the assertions stated in the exercises. (You may always use things mentioned in class, problems from previous homeworks, and earlier problems from the same homework.)

- I. How do the axioms for hyperbolic geometry differ from the axioms for Euclidean geometry? For any axiom that is different, write down the hyperbolic-geometry version of the axiom.
- II. Let D_M denote the diameter of the moon, D_S the diameter of the sun, R_M the distance from the earth to the moon, and R_S the distance from the earth to the sun. Explain, including a diagram, how a solar eclipse (when the moon passes in front of the sun, as seen from the earth) gives an equation relating these four quantities.
- III. For this question, "draw" means "draw, with reasonable accuracy, a picture using the disk model of the hyperbolic plane". *For this question, do not allow any of your hyperbolic lines to look like diameters of the boundary Euclidean circle.*
 - (a) Draw three lines *L*, *M*, and *N* such that *L* and *M* are parallel to each other, *L* and *N* are ultraparallel to each other, and *M* and *N* intersect each other. (Be sure to label the lines correctly.)
 - (b) Draw a quadrilateral *ABCD* such that each of its four angles is less than 30°. (Be sure that all four vertices really are points in the hyperbolic plane.)
 - (c) Draw a line *L* through a point *P*, and draw the image of *L* under the isometry (of the hyperbolic plane) that is a rotation around *P* through a 120° angle. Mark the 120° angle in your drawing, and label the line *L* to distinguish it from its image line.
- IV. One of the two angles 2° and 3° is constructible using a straightedge and compass, while the other one is not. Which one is which? Explain your answers. (Hint: relate the two angles to appropriate regular polygons.)
- V. Let *A* and *B* be already constructed points, and let C be an already constructed circle that is centered at *A*. Suppose that the point *B* is inside the circle *C*. Describe how to construct, with straightedge and compass, a circle that goes through *B* and is tangent to C.

VI. Describe how to construct, with straightedge and compass, an angle θ such that

$$\sin\theta = \frac{\sqrt{3} + \sqrt{5}}{7}.$$

You may use the fact that $0 < \sqrt{3} + \sqrt{5} < 7$ without having to prove it. You do not have to describe each individual line and circle drawn, as long as you indicate the steps you are taking and why they are possible.

VII. In hyperbolic geometry, suppose that the line \mathcal{L} intersects two other lines \mathcal{M} and \mathcal{N} in such a way that the two alternate interior angles are equal. Prove that the lines \mathcal{M} and \mathcal{N} are ultraparallel. (Do not use facts that were established only in the course of proving this statement before.)