Math 308, Section 101
Second Midterm Exam
November 10, 2004
Duration: 50 minutes

Name: ______________________ Student Number: ______________________

Do not open this test until instructed to do so! This exam should have 9 pages, including this cover sheet. No textbooks, notes, calculators, or other aids are allowed. Turn off any cell phones, pagers, etc. that could make noise during the exam. You must remain in this room until you have finished the exam.

All your solutions must be written clearly and understandably. Use complete sentences and explain why your mathematical statements are relevant to the problem. You should always write enough to demonstrate that you're not just guessing the answer. Use the backs of the pages if necessary. You will find some of the questions quite easy; try to solve these first. Good luck!

Problem	Out of	Score
1	6	
2	6	
3	8	
4	8	

Problem	Out of	Score
5	6	
6	8	
7	8	
Total	50	

These facts are here for your reference. This is not a complete list; you may quote any facts we have covered, as long as you make it clear what fact you are using. You are not required to quote them by theorem number, lemma number, and so on (but you may if you wish to). **Please do not remove this page from the test booklet.**

BOTH EUCLIDEAN GEOMETRY AND HYPERBOLIC GEOMETRY

Angles forming a straight line add to 180°, and vertical angles are always congruent.

The **SSS**, **SAS**, and **ASA** triangle congruence theorems hold.

Pons Asinorum. The base angles of an isosceles triangle are equal: if $|AB| = |AC|$, then $\angle ABC = \angle ACB$. [Note: the converse is also true: if $\angle ABC = \angle ACB$, then $|AB| = |AC|$.]

EUCLIDEAN GEOMETRY ONLY

Star Trek Lemma. The measure of an inscribed angle equals half of the angular measure of the arc it subtends. [Note: the angular measure of an arc PQ on a circle centered at O is equal to the measure of the central angle $\angle POQ$.]

Bow Tie Lemma. Let A, A', B, and C lie on a circle, and suppose that $\angle BAC$ and $\angle BA'C$ subtend the same arc. Then $\angle BAC = \angle BA'C$.

Power of the Point Theorem. Let P be any point and $\mathcal{C}$ any circle, and let ℓ_1 and ℓ_2 be two lines through P. Suppose that ℓ_1 intersects $\mathcal{C}$ in the two points Q and Q', and ℓ_2 intersects $\mathcal{C}$ in the two points R and R'. Then $|PQ||PQ'| = |PR||PR'|$.

Heron's Formula. Let s be the semiperimeter of $\triangle ABC$. Then the area of $\triangle ABC$ is given by the formula $|\triangle ABC| = \sqrt{s(s-a)(s-b)(s-c)}$. [Note: a, b, and c are the lengths of the sides of $\triangle ABC$, and $s = \frac{1}{2}(a+b+c)$.]

Law of Cosines. For any triangle $\triangle ABC$, we have $c^2 = a^2 + b^2 - 2ab\cos C$.

Extended Law of Sines. Let R be the circumradius of $\triangle ABC$. Then $a/\sin A = b/\sin B = c/\sin C = 2R$. [Note: the circumradius of a triangle is the radius of the circle that can be circumscribed around it.]

Fact about cubics. Let $P(x) = ax^3 + bx^2 + cx + d$ be an integer polynomial. Suppose that $P(r) \neq 0$ for every number r of the form $r = \pm m/n$, where m divides d and n divides a. Then $P(x)$ is irreducible over the integers.

Constructibility questions. It **is** possible to trisect an arbitrary line segment using straightedge and compass. However, it is **not** possible to trisect an arbitrary angle using a straightedge or compass, nor to double the cube, nor to square the circle.

Theorem 3.6.5. Let p be an odd prime. For any integer $r \geq 2$, it is impossible to construct a regular p^r-gon. Furthermore, a regular p-gon is constructible if and only if p is of the form $2^{2^k} + 1$. [Note: these special primes p are called Fermat primes.]

HYPERBOLIC GEOMETRY ONLY

Theorem 6.3.1. If two triangles are similar, then they are automatically congruent.

Theorem 6.4.2. Two ultraparallel lines have a common perpendicular.

Let Q be a point on a line ℓ, and let P be a point such that PQ is perpendicular to ℓ. Let PR be a line through P that is parallel to ℓ. Then $\angle QPR$ is called the **angle of parallelism**. The angle of parallelism is always less than 90°, and it depends only on the length $|PQ|$. If PR' is a line through P such that $0^\circ \leq \angle QPR' < \angle QPR$, then the line PR' intersects ℓ; on the other hand, if PR'' is a line through P such that $\angle QPR < \angle QPR'' \leq 90^\circ$, then the line PR'' is ultraparallel to ℓ.