Math 312, Section 102
Homework #1
due Tuesday, September 18, 2001 at the beginning of class

I. Let a and b be positive integers. Show that among all numbers of the form $a - bk$ where k is an integer, there is one that is the smallest positive integer of that form. (This is the same as Rosen, Section 1.1, p. 14, #2; I've just tried to word it a little more clearly.)

II. Let a and d be any numbers. Prove that

$$\sum_{j=0}^{n}(a + jd) = (n + 1)\Big(a + \frac{nd}{2}\Big).$$

(Here is a good way to remember this formula, once you've proved it: the sum of the numbers in an arithmetic progression equals the number of terms times the average of the first and last terms.)

III. Rosen, Section 1.2, p. 22, #3 and #13

IV. Rosen, Section 1.2, p. 23, #20

V. Rosen, Section 1.3, p. 28, #10

VI. Define $\alpha = \dfrac{1 + \sqrt{5}}{2}$ and $\beta = \dfrac{1 - \sqrt{5}}{2}$.
(a) Verify the two equalities $\alpha + 1 = \alpha^2$ and $\beta + 1 = \beta^2$.
(b) Use the second principle of induction to prove Theorem 1.4.