Math 331, Section 201—Homework #1
due in class Tuesday, January 21, 2003

Warm-Up Questions—do not hand in

Graham, Knuth, and Patashnik, Chapter 2, pp. 63–65, #5–7, #9, #14, #17, #19, #23, #29

Homework Questions—turn in solutions to these problems

I. Use summation by parts to evaluate

$$\sum_{0 \le k < n} \frac{k}{(k+1)(k+2)(k+3)}.$$

II. (a) Prove that $\Delta(Eu) = E\Delta(u)$ for any function $u(x)$.

(b) Given a function $u(x)$ and a positive integer m, let u_m be the function defined by $u_m(x) = u(mx)$. Prove that

$$\Delta(u_m)(x) = \sum_{0 \le k < m} \Delta(u)(mx+k).$$

III. Find a closed-form formula for the sequence T_n defined by the following recurrence:

$$\begin{aligned} T_1 &= 2 \\ \frac{2T_n}{n} &= \frac{T_{n-1}}{n+1} + 1 \quad (n \ge 2). \end{aligned}$$

IV. Given two functions $u(x)$ and $v(x)$, find a formula for $\Delta\left(\frac{u(x)}{v(x)}\right)$. Justify your answer. Use your formula to calculate

$$\Delta\left(\frac{H_x}{3^x+1}\right).$$

V. Let m be a positive integer. Find a formula for

$$\sum_{1 \le j < k \le n} \frac{j^{\overline{m}}}{k^{\overline{m}}}$$

in terms of m and n. (Hint: don't forget that rising factorial powers are related to falling factorial powers. Does it look easier to do the sum over j first or the sum over k first?) Find an integer m such that

$$\sum_{1 \le j < k \le 20} \frac{j^{\overline{m}}}{k^{\overline{m}}} = 1.$$