Math 331, Section 201—Homework #2
due in class Tuesday, February 4, 2003

Warm-Up Questions—do not hand in

Graham, Knuth, and Patashnik, Chapter 5, pp. 242–246, #2–4, #7, #8, #13, #17, #37, #41, and #45

Homework Questions—turn in solutions to these problems

I. Given a fixed real number s, calculate $\Delta\big((-1)^n\binom{s}{n}\big)$ as a function of n. Then use the Fundamental Theorem of Finite Calculus to show that for all real numbers r and all integers m,

$$\sum_{k\le m}(-1)^k\binom{m+r}{k}=\binom{-r}{m}.$$

II. Given positive integers m and n, evaluate the double sum

$$\sum_{0\le j\le m}(-1)^j\sum_{0\le k\le n}\binom{m-k}{j}.$$

III. Given integers m and n, evaluate the sum

$$\sum_{k\le m}\binom{n+k}{n-m}\binom{m+k}{k}$$

IV. Assuming equation (5.23) on page 169 of Graham, Knuth, and Patashnik, derive equation (5.24) by using manipulations like those in Table 174. (Is the derivation easier if s is an integer?)

V. Given a fixed real number r and a fixed integer n, evaluate the following double sum over k and m:

$$\sum_{k\le m}\binom{n}{n-m}\binom{r}{k}\Big(1-\frac{2k}{r}\Big)$$

VI. (a) Prove that

$$S_n=\sum_{k\le n}\binom{n-k}{k}2^k=\frac{1}{3}\big(2^{n+1}+(-1)^n\big)$$

for all nonnegative integers n. (Hint: first prove that $S_n=S_{n-1}+2S_{n-2}$ when $n\ge 2$. Then define a new sequence $T_n=S_n-2S_{n-1}$, and find a formula for T_n. Then use a summation factor to convert this into a formula for S_n.)

(b) Use part (a) to evaluate

$$\sum_{k<n}\frac{n}{n-k}\binom{n-k}{k}2^k$$

for all positive integers n.