Formulas

On the exam itself, I'll add a few more words to help indicate what the notations mean. But these will be the formulas included with the exam.

If $\{a_n\} \stackrel{ops}{\longleftrightarrow} f(x)$ then, for $h \geq 1$,

$$\{a_{n+h}\} \stackrel{ops}{\longleftrightarrow} \frac{f(x) - a_0 - a_1 x - \cdots - a_{h-1}x^{h-1}}{x^h} \qquad \left\{ \sum_{n_1+\cdots+n_k=n} a_{n_1} a_{n_2} \cdots a_{n_k} \right\} \stackrel{ops}{\longleftrightarrow} f(x)^k$$

If $\{b_n\} \stackrel{egf}{\longleftrightarrow} g(x)$ then, for $h \geq 1$,

$$\{b_{n+h}\} \stackrel{egf}{\longleftrightarrow} D^h g(x)$$

$$\sum_n \binom{\alpha}{n} x^n = (1+x)^\alpha \qquad \sum_n \binom{n+k}{n} x^n = \frac{1}{(1-x)^{k+1}}$$

$$\sum_n \frac{1}{n+1}\binom{2n}{n} x^n = \frac{1-\sqrt{1-4x}}{2x} \qquad \sum_n \binom{2n}{n} x^n = \frac{1}{\sqrt{1-4x}}$$

$$\sum_{n\geq 0} \frac{k(2n+k-1)!}{n!(n+k)!} x^n = \left(\frac{1-\sqrt{1-4x}}{2x}\right)^k \qquad \sum_{n\geq 1} \frac{x^n}{n} = \log\frac{1}{1-x}$$

$$\mathcal{H}(x,y) = e^{y\mathcal{D}(x)}$$

$$h(n,k) = \left[\frac{x^n}{n!}\right]\left\{\frac{\mathcal{D}(x)^k}{k!}\right\}$$

$$\sum_{n\geq 0}\sum_{k\geq 0} p(n,k)x^n y^k = \prod_{r\geq 1}\frac{1}{1-yx^r}$$

$$\mu = \frac{F'(1)}{F(1)} \qquad \sigma^2 = \left\{(\log F)' + (\log F)''\right\}\Big|_{x=1}$$

$$N_r = \sum_{S:\ \#S=r} N(\supset S) \qquad E(x) = N(x-1)$$

$$N_r = \sum_{t\geq 0}\binom{t}{r}E_t \qquad E_t = \sum_{r\geq 0}(-1)^{r-t}\binom{r}{t}N_r$$

Let C be a (strangely shaped) chessboard contained in an $n \times n$ board. If r_k is the number of ways of placing k non-attacking rooks on C, then the number of permutations of $\{1, 2, \ldots, n\}$ that hit C in exactly j squares is

$$[x^j]\sum_k (n-k)!\,r_k(x-1)^k.$$

Lagrange Inversion: If f, ϕ, and u are power series in t with $\phi(0) \neq 0$ and $u = t\phi(u)$, then

$$[t^n]\{f(u(t))\} = \frac{1}{n}[u^{n-1}]\{f'(u)\phi(u)^n\}.$$

$$PP(f; z_0) = \sum_{j=1}^{r} \frac{a_{-j}}{(z-z_0)^j} = \sum_{n\geq 0} z^n \left\{ \sum_{j=1}^{r} \frac{(-1)^j a_{-j}}{z_0^{n+j}} \binom{n+j-1}{j-1} \right\}.$$

$$[z^n]f(z) = [z^n]\left\{ \sum_{k=0}^{s} PP(f; z_k) \right\} + O\left(\left(\frac{1}{R'} + \varepsilon \right)^n \right)$$

$$[z^n]\{(1-z)^\beta v(z)\} = \sum_{j=0}^{m} v_j \binom{n-\beta-j-1}{n} + O\big(n^{-m-\beta-2}\big)$$

If $f(z)$ is admissible, $a(r) = rf'(r)/f(r)$, $b(r) = ra'(r)$, $a(r_n) = n$, then

$$[z^n]f(z) \sim \frac{f(r_n)}{r_n^n \sqrt{2\pi b(r_n)}}.$$

If $p(z)$ is a polynomial with $[z^1]p(z) \neq 0$, then $e^{p(z)}$ is admissible.