Math 331–Homework #1
due at the beginning of class Monday, January 15, 2007

Reality-check problems. Not to write up; just ensure that you know how to do them.

I. Wilf, p. 24, #1.1 and 1.3

II. Wilf, p. 25, #1.5 (a), (d), and (e)

III. Wilf, p. 25, #1.6 (b) and (c)

IV. Suppose that $f(x) = \sum_{n=0}^{\infty} a_n x^n$ and $g(x) = \sum_{n=0}^{\infty} b_n x^n$ are power series. Show that $[x^n]\big(f(x)g(x)\big) = \sum_{k=0}^{n} a_k b_{n-k}$.

Homework problems. To write up and hand in.

I. Wilf, p. 25, #1.9, except you don't have to write up the "and therefore that" part of the problem.

II. Wilf, p. 26, #1.11

III. Wilf, p. 27, #1.15

IV. Wilf, p. 27, #1.16

V. Wilf, p. 27, #1.17

VI. Wilf, p. 27, #1.18 (a) and (c). In part (c), explicitly include the values of n and k for which $f(n,k)$ is defined, and the values of n and k for which the recurrence is valid. If you wish to define $f(n,k)$ to be 0 for certain n and k, you're welcome to do so—just be explicit. Note: it's more standard to call the numbers $f(n,k)$ *Eulerian numbers*, rather than Euler numbers.

VII. Using the values of the first several Eulerian numbers given at
`http://mathworld.wolfram.com/EulerianNumber.html`,
calculate all values of $f(8,k)$, where f is defined in the previous problem.

VIII. (a) Show that $\binom{-1/2}{n} = \left(-\frac{1}{4}\right)^n \binom{2n}{n}$ for all integers n.

(b) Using part (a) and (1.5.5) in Wilf, show that $\frac{1}{\sqrt{1-4x}}$ can be written as $\sum_n \binom{2n}{n} x^n$.

(c) Using part (b) and reality-check IV, prove that $\sum_k \binom{2k}{k}\binom{2n-2k}{n-k} = 4^n$.

IX. Consider two strangely-labeled six-sided dice: the red one has the numbers 1, 2, 2, 3, 3, and 4 on its faces, while the blue one has the numbers 1, 3, 4, 5, 6, and 8 on its faces. Using generating functions, show that when rolling this pair of dice and adding the values, the various results occur exactly as often as they would with a pair of normal six-sided dice.