Math 331–Homework #2
due at the beginning of class Monday, January 29, 2007

Reality-check problems. Not to write up; just ensure that you know how to do them.

I. Wilf, p. 24, #2 and 4
II. Wilf, p. 25, #5 (b) and (c)
III. Wilf, p. 65, #1, 2, and 4

Homework problems. To write up and hand in.

I. Wilf, p. 25, #7
II. Wilf, p. 26, #10(e)
III. Wilf, p. 28, #21(a)
IV. Wilf, p. 65, #6
V. Wilf, p. 67, #22
VI. Wilf, p. 69, #27
VII. Find a closed form for the exponential power series generating function $\sum_n F_n^2 x^n/n!$ of the squares of the Fibonacci numbers.
VIII. Let $T_k(x) = \sum_n \left\{ {n \atop k} \right\} \frac{x^n}{n!}$ be the exponential power series generating function for the Stirling numbers of the second kind.
  (a) Show that $T_k(x)$ satisfies the equation $T_k'(x) = kT_k(x) + T_{k-1}(x)$ for $k \geq 1$.
  (b) Usng part (a) or otherwise, prove that
$$T_k(x) = \frac{(e^x - 1)^k}{k!} \quad \text{for } k \geq 0.$$
  (c) Using part (b) or otherwise, prove the identity
$$\left\{ {n+1 \atop m+1} \right\} = \sum_k \binom{n}{k} \left\{ {k \atop m} \right\}.$$
IX. Consider the formal power series e^x.
  (a) Show that $e^{2x} = e^x e^x$, where the left-hand side is the composition of the two formal power series e^x and $2x$, while the right-hand side is the product of two formal power series.
  (b) Why is it not correct to use the same technique to argue that $e^{C+x} = e^C e^x$ as formal power series, when C is a constant?