

## Math 432/537 Homework #1

due Friday, September 20, 2002 at the beginning of class

**Homework policies:** You are permitted to consult one another concerning the homework assignments, but your submitted solutions must be written by you in your own words. I will consider not only correctness but also clarity when evaluating your work. You should attempt all of the problems, but if you consistently solve 75% of the homework problems correctly or with only minor errors, you will earn at least an A– for your homework mark. Do endeavor to understand later how to solve any problems you omit.

- I. Niven, Zuckerman, and Montgomery, Section 1.2, p. 20, #53
- II. Niven, Zuckerman, and Montgomery, Section 1.3, pp. 29–31, #14, #16, and #26
- III. Niven, Zuckerman, and Montgomery, Section 2.1, pp. 57–59, #20, #25, and #54
- IV. Using only the ten axioms for the set of integers given on the attached page, prove the following. Explicitly mention when you are using axioms.
  - (a)  $a^2 \geq 0$  for every integer  $a$ , with  $a^2 = 0$  if and only if  $a = 0$ .
  - (b) There is no integer  $x$  such that  $0 < x < 1$ .
- V. Let  $x$  be a rational number. Prove that there are unique integers  $m$  and  $n$  such that  $(m, n) = 1$ ,  $n > 0$ , and  $m/n = x$ .
- VI. Prove that if  $a$  is a divisor of  $b$  and  $a > (b, c)$ , then  $a$  is not a divisor of  $c$ .
- VII. Let  $a$  and  $b$  be integers, not both zero. Prove that  $(a, b) = (a + b, [a, b])$ .
- VIII. Do this problem by hand, showing your work! Calculate  $(3724, 817)$ , and find integers  $x$  and  $y$  such that  $3724x + 817y = (3724, 817)$ . Calculate  $[3724, 817]$ .
- IX. Suppose that  $(a, b) = 1$  and that  $a \mid n$  and  $b \mid n$ . Prove that  $ab \mid n$ .
- X. Prove that  $((a, b), c) = (a, (b, c))$  for all positive integers  $a, b, c$ .
- XI. The *Fibonacci numbers* are defined recursively by  $F_0 = 0$ ,  $F_1 = 1$ , and  $F_n = F_{n-1} + F_{n-2}$  for all  $n \geq 2$ . The first several Fibonacci numbers are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,  $\dots$ , so that  $F_{12} = 144$ , for example.
  - (a) Let  $k$  be a positive integer. Prove that  $F_{m+k} \equiv F_{k+1}F_m \pmod{F_k}$  for every  $m \geq 0$ .
  - (b) Using part (a), or otherwise, prove that if  $k \mid n$  then  $F_k \mid F_n$ .