**Math 432/537 Homework #5**

due Friday, November 15, 2002 at the beginning of class

I. Niven, Zuckerman, and Montgomery, Section 5.3, p. 233, #9

II. Prove the following theorem of Dickson from 1894: $(x, y, z)$ is a primitive Pythagorean triple if and only if $(x, y, z) = (\rho+\tau, \sigma+\tau, \rho+\sigma+\tau)$ for some positive integers $\rho, \sigma, \tau$ satisfying $(\rho, \sigma) = 1$ and $\tau^2 = 2\rho\sigma$. Given such a triple $(\rho + \tau, \sigma + \tau, \rho + \sigma + \tau)$, how can we recover the values of $r, s, t$ that give the representation in Theorem 5.5?

III. Niven, Zuckerman, and Montgomery, Section 5.4, pp. 239–240, #4, #6, #7, and #10. It might be useful for you to prove the following fact: if $x$ is a quadratic residue (mod 7) then $x^3 \equiv 1 \pmod 7$, while if $x$ is a quadratic nonresidue (mod 7) then $x^3 \equiv -1 \pmod 7$.

IV. Niven, Zuckerman, and Montgomery, Section 5.6, p. 260, #5. In addition, find two nonzero rational numbers $x, y$ such that $y^2 = x^3 + 2x^2$ and both $|x|$ and $|y|$ are less than $\frac{1}{100}$.

V. Let $g(x, y) = x + 2x^2 + 3x^3 + 3y + y^3$.

(a) Two points such that $g(x, y) = 10$ are $(1, 1)$ and $(-3, 4)$. Find a third point $(u, v)$ with rational coordinates such that $g(u, v) = 10$ by considering the line passing through the first two points.

(b) Find two points $(u, v)$ with rational coordinates such that $g(u, v) = 0$. One such point should be obvious; find the second by considering the tangent line to the curve $g(x, y) = 0$ passing through the first point.

VI. *Bonus problem*: Niven, Zuckerman, and Montgomery, Section 5.3, p. 234, #15

VII. Niven, Zuckerman, and Montgomery, Section 7.2, p. 329, #1

VIII. Calculate the entire infinite continued fraction expansion of $1 + \sqrt{3}/5$. Calculate the real number whose continued fraction expansion is $\langle 4, 3, 2, 1, 3, 2, 1, 3, 2, 1, \dots \rangle$.

IX. Calculate enough of the continued fraction of $e = 2.71828\dots$ to make a conjecture about the entire infinite continued fraction expansion of $e$. Show your work.

X. Calculate the 0th, 1st, 2nd, 3rd, and 4th convergents to $\pi$, showing your work.