Math 432/537 Homework #6
due Friday, November 29, 2002 at the beginning of class

I. Niven, Zuckerman, and Montgomery, Section 7.3, p. 333, #4

II. (a) Niven, Zuckerman, and Montgomery, Section 7.3, p. 333, #6
   (b) Using the method in part (a) and the fact that $17682^2 \equiv -1 \pmod{100049}$, find a representation of the prime 100049 as the sum of two squares, showing your work.

III. Niven, Zuckerman, and Montgomery, Section 7.4, p. 336, #6

IV. Niven, Zuckerman, and Montgomery, Section 7.5, p. 340, #3 and #4

V. (a) Niven, Zuckerman, and Montgomery, Section 7.5, p. 341, #6
   (b) Suppose that $\frac{1}{2} < x < 1$ and that $r_0, r_1, r_2, \ldots$ are the convergents to x. Show that for all $n \geq 0$, the nth convergent to $1 - x$ is $1 - r_{n+1}$.

VI. Niven, Zuckerman, and Montgomery, Section 7.6, p. 344, #3

VII. Niven, Zuckerman, and Montgomery, Section 7.8, p. 357, #9

VIII. The numbers ξ_n arising while computing the continued fraction expansion of ξ_0 can be represented as $\xi_n = (t_n + u_n \xi_0)/(v_n + w_n \xi_0)$ for some integers t_n, u_n, v_n, w_n depending on ξ_0. For example, $\xi_1 = 1/(\xi_0 - a_0)$, so we can take $t_1 = 1, u_1 = 0, v_1 = -a_0, w_1 = 1$. Find and prove formulas for t_n, u_n, v_n, w_n (in terms of quantities like a_n, h_n, k_n).

IX. Prove that for every real number x and every positive integer n, there exists a rational number $\frac{a}{b}$ with $0 < b \leq n$ and $|x - \frac{a}{b}| \leq \frac{1}{b(n+1)}$.

X. (a) Show that the equation $x^2 + x = 2y^2$ has infinitely many integer solutions. (Hint: complete the square.) Then solve Niven, Zuckerman, and Montgomery, Section 7.8, p. 357, #5.

   (b) In Triangle Town, the streets are named 2nd Avenue, 3rd Avenue, 4th Avenue, and so on. There are exactly n houses on nth Avenue, the numbers of which (from west to east) are $1, 2, \ldots, n$. Alice, Bob, and Eve have the following conversation:

   BOB: What street do you live on, Alice?
   (Alice whispers the answer to Bob so that Eve does not hear)
   BOB: What's the number of your house, Alice?
   ALICE: I won't tell you the number, but I will tell you how to find my house. Start at one end of my street and walk down it, adding up the numbers of all the houses one by one as you come to them. When your running total equals the following number ... *(Alice whispers a number to Bob so that Eve does not hear)* ... then my house is the last house you added in. *(Alice starts to walk away)*
   BOB: Wait, Alice: which end of your street do I start from, the west end or the east end?
   ALICE: That's the best part: it doesn't matter! *(Alice leaves)*
   EVE: I know you don't want to tell me Alice's address, Bob, but at least tell me this much: how many digits are there in her street number?
   (Bob answers Eve's question)

   What is Alice's address (street and house number) if Bob's answer was "One digit"? if Bob's answer was "Two digits"? if Bob's answer was "Three digits"?

XI. BONUS: Niven, Zuckerman, and Montgomery, Section 7.6, p. 344, #4