(your name goes here)

Math 539—Homework #0
due Wednesday, January 10, 2018 by 11:59 AM

Homework policies: You are permitted to consult one another concerning the homework assignments, but your submitted solutions must be written by you in your own words. I will consider not only correctness and rigor but also clarity when evaluating your work.

The purpose of this homework is to make sure you have access to LaTeX and have practiced a little bit of typesetting. **For this homework only**, please email me your TeX file as well as the PDF file it generates. Remember to prepend your name to the filename—use the format `GregMartin-homework0.tex` and `GregMartin-homework0.pdf`.

When you first compile this TeX file, the output won't look exactly like the `homework0.pdf` you downloaded: certain images that were included in `homework0.pdf` for problems #4, 5, 7, 8, and 9 won't appear in your output. Note that the TeX code for those problems (below) has commands that have been commented out using the % symbol (all such suppressed commands begin with `\image`). If you uncomment the commands by removing the %, this TeX file won't compile because you don't have the required image files. You can simply remove those `\image` commands entirely (or leave the comments in place, if you want), and use the PDF file you downloaded as a visual guide to the tasks you're being asked to perform.

This homework isn't a complete from-the-ground-up introduction to TeX but rather a guide to some of the common things you'll need to know to use it. Some of what you can learn from this file isn't spelled out explicitly in the questions you're answering, but rather by having this TeX file and the corresponding output to compare and adapt to future need. (For example, you see from reading this TeX file that math content needs to occur between dollar signs or inside some other sort of math-making environment.)

Simple relations are easy to typeset straight from the keyboard, while other relations can be typed with TeX commands (which start with a backslash). Here are four expressions joined by valid relation symbols:
$$1 < 1 + 1 = 2 \leq 2 + x^2.$$

1. *Write those four expressions in the reverse order, with relation symbols between them that give the same information as the above line.*

(your solution goes here)

We use the divisibility relation a lot: $3 \mid 12$ and $7 \mid 91$. Notice that `\mid` makes the spacing nicer than using the $|$ symbol from the keyboard (compare to $7|91$).

Most relations can be negated by putting `\not` before them: $-1 \not< -2$ and $1 \not\cong 2$. A couple of commonly negated symbols have special names: `\ne` and `\nmid`. (The first one just allows you to type fewer characters. The second one is because the `\mid` character is so thin that `\not\mid` looks awful: $3 \not| 7$.)

2. *Each of these statements is incorrect; edit them to be correct by negating the relations.*

    (a) $1 \geq 2$
    (b) $\pi \approx 4$
    (c) $5 = 6$
    (d) $7 \mid 8$
    (e) $9 > 10$
    (f) $11 \equiv 12$

You can produce Greek letters by putting a backslash before their name, like ω. If the capital version doesn't look exactly like a Roman letter, you can product it similarly, like Ω.

3. *The first seven letters of the Greek alphabet are alpha, beta,[1] gamma, delta, epsilon, zeta, and eta. Produce both lowercase and uppercase versions of all seven. (A bare string of those 14 letters is fine.)*

(your solution goes here)

TEX contains way more symbols than we will ever use. If you want a particular symbol but don't know the backslash-command that produces it, one fun way to find out is to use the website: `http://detexify.kirelabs.org/classify.html`

4. *Replace each image below by the TEX command that typesets the same symbol.*

    (a) ‡
    (b) ↩
    (c) $\amalg$

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[1]Yes, that is where the English word "alphabet" comes from! By the way, look at the TEX file to see how to make smart quotes; as you'll find, "this" doesn't work, but 'this' does.

Putting an expression (however complicated) inside curly brackets makes TEX treat it as a single thing. Most commonly, you'll use that with superscripts, subscripts, square roots, and fractions. For example, a superscript raises a single thing, like x^2; but curly brackets can make that superscripted thing as complicated as you wish, like x^{3y^4+5}.

Two random items: First, you can turn square roots into higher roots, like $\sqrt[30]{t}$, by putting the appopriate expression in square brackets. Second, TEX treats the prime $'$ in x' as special (to save typing): you shouldn't include the $\hat{}$ explicitly—the prime includes it in its definition. (Notice that $x^{'}$ looks funny compared to x'.)

5. *Replace each image below by the TEX commands that typeset the same expression.*

    (a) $\sqrt{12}$
    (b) $T_{\pm 1}$
    (c) $\frac{f(x)}{g(x)}$
    (d) $\sqrt[3]{\mu_0'}$

Most common math functions are typeset better by starting their names with a backslash; for example, compare $\log a + \log b = \log ab$ with $loga + logb = logab$. (This example also suggests, correctly, that TEX ignores redundant white space and returns when it processes a TEX file.)

6. *Complete the following sentence.*

My favorite trigonometric identity is (your solution goes here).

We'll use congruences (mod m) a lot in this course; but our convention is to use the `\pmod` command, which automatically includes parentheses, instead of the `\mod` command. To insert text into math formulas beyond names of functions, just use the `\text` command.

7. *Replace the image below by the TEX commands that typeset, as a displayed equation, the same mathematical sentence. Make sure the spacing around the word "whenever" is sensible.*

$$a^{\phi(m)} \equiv 1 \pmod{m} \text{ whenever } \gcd(a,m) = 1.$$

"Delimiters" are bracket-type objects that typically surround math expressions, such as the angle brackets in $y + \langle a^b \rangle + z$. (Note how that spacing is much better than $y+ < a^b > +z$.) Often the delimiters will need to get bigger to appropriately contain the expression inside. You can use the `\left/\right` or `\big/\bigg` delimiter modifiers to do this. For more information, see: https://en.wikibooks.org/wiki/LaTeX/Mathematics#Brackets.2C_braces_and_delimiters

8. *Replace the image below by the T$_E$X commands that typeset, as a displayed equation, the same mathematical monstrosity. (Just get the delimiter sizes roughly right—don't worry about exactly.)*

$$\left(\left\lceil \left\{ \left[a^b \right\rfloor^2 \right\| ^3 \right\rceil ^4 \right.$$

Sums can be written with lots of combinations of lower and upper limits/conditions of summation:

$$\sum_n f(n) \quad \sum^{\infty} f(n) \quad \sum_{j=1}^{k} x^j \quad \sum_{\substack{j=1 \\ j \neq i}}^{k} x^j \quad \sum_{\substack{d|n \\ d \leq \sqrt{n} \\ d \text{ prime}}} \phi(d)$$

(Note the use of `\substack` in the last two.) Products are similar, using `\prod`. By default the limits look a little different when typeset inline: $\prod_{j=1}^{k} x^j$.

9. *Replace the image below by the T$_E$X commands that typeset, as a displayed equation, the same mathematical equation—except fill in the ?? on the right-hand side in a way that makes it mathematically correct.*

$$\prod_{n=1}^{\infty} (1 - p_n^{-s})^2 = \exp\left(\sum_{n=1}^{\infty} ?? \right).$$

There are several ways to get tables in L^{A}T$_E$X. One way is using the `\tabular` environment:

2	$x+1$	x^2+1	x^3+1
$1+x$	$2x$	x^2+x	x^3+x
$1+x^2$	$x+x^2$	$2x^2$	x^3+x^2

10. *Typeset the above table, but without any horizontal or vertical lines separating the entries, and with all entries centre-justified instead of right-justified or left-justified, and with one extra row and column that continue the given mathematical pattern.*

(your solution goes here)

If a displayed equation is too long, it can be split using the `multline` or `multline*` environments (the former gives it an equation number, the latter doesn't).

11. *Edit the following long displayed equation to split it onto three lines using* `multline`.

$$\det\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} = afkp-aflo-agjp+agln+ahjo-ahkn-bekp+belo+bgip-bglm-bhio+bhkm+cejp- \tag{1}$$

If instead you want the multiple lines to line up at a certain point (a running equals sign, for example), then use the `align` environment. (The `split` environment is closely related, if you want to look into that.)

12. *Edit the following aligned system of congruences to correct the typesetting and alignment.*

$$a \equiv b \ (\mathrm{mod}\ 4)$$
$$c \equiv d \ (\mathrm{mod}\ 16)$$
$$e \equiv f \ (\mathrm{mod}\ 6)4g \qquad\qquad \equiv h \quad \mathrm{mod}\ 256$$

Finally, your mathematical expressions and statements should be parts of grammatical sentences: the capitalization and punctuation should be the same as if you were reading the expressions aloud. For example, the polynomial $(x+3)^2$ expands into $x^2 + 6x + 9$, while the polynomial $x^2 + 1$ is irreducible. (Part (a) of the next problem gives another example.) Punctuation should be outside the math environment when possible (for better spacing), although this rule has to be broken in displayed equations like

$$(x+3)^2 = x^2 + 6x + 9.$$

13. *The parts of this question are a little open-ended; don't stress—just answer thoughtfully.*

(a) *Write the computation*

$$\langle x_j, c_1 x_1 + \cdots + c_n x_n \rangle = c_1 \langle x_j, x_1 \rangle + \cdots + c_n \langle x_j, x_n \rangle \tag{2}$$
$$= c_j \langle x_j, x_j \rangle + \big(c_1 \langle x_j, x_1 \rangle + \cdots + c_{j-1} \langle x_j, x_{j-1} \rangle + c_{j+1} \langle x_j, x_{j+1} \rangle + \cdots + c_n \langle x_j, x_n \rangle \big)$$
$$= c_j \cdot 1 + (0 + \cdots + 0 + 0 + \cdots + 0) = c_j \quad (j = 1, \ldots, n)$$

using summation notation (and no dot-dot-dots). Make sure that only the last line of your version of the computation has an equation number.

(b) *Notice that some of the dot-dot-dots in equation (2) (and below) are floating above the baseline, while others are sitting right on the baseline. (And notice the T_EX commands that produce that difference.) When do you think you should use one or the other, in general?*

(c) *What linear algebra theorem would contain the above computation in its proof? Get L^AT_EX to make a reference to your equation number in your answer.*

(a) (your solution goes here)
(b) (your solution goes here)
(c) (your solution goes here)