TITLE OF THE ARTICLE

YOUR NAME

ABSTRACT. Here's a LaTeX template.

1. INTRODUCTION

The content of this manuscript is not interesting. The purpose of this manuscript is to provide a template for using LaTeX to complete your written assignment for MATH 613E: Topics in Analytic Number Theory. It can also help as a general LaTeXprimer. For example, look at this sentence in the file `template.tex` to see that LaTeX mostly ignores white space. (If you're keen, look closely at the `.tex` file to figure out why the "LaTeX" got stuck to the word "primer" in the 3rd sentence of this article, but was fine in the sentence before that. Now look at the sentence before this one to see how LaTeX processes quotation marks.) In general, comparing the output to the `.tex` file will teach you (if you don't already know) how do to all sorts of things.

A blank line in the `.tex` file makes a new paragraph. Don't try to do line breaks or page breaks yourself; LaTeX does that automatically.

You can *emphasize parts of the text*. If you really want to control how text looks, you can *put things in italics* or **in boldface** or in a `fixed-width font`.

You can make comments in the `.tex` file that don't appear in the output, if it helps you. Between this sentence and the previous one is one such comment.

In Section 2, we'll talk about typesetting mathematics; in Section 3, which starts on page 3, we'll talk about how to automatically number equations, theorems, and sections (some automatically generated numbers appear in this very sentence). Section 4 discusses macros, which are definitions of your own commands. Finally, in Section 5 we'll suggest a format for your writing assignment for MATH 613E: Topics in Analytic Number Theory.

Feel free to experiment with changing your own copy of this file to see what happens; an original copy will remain on the course web page should you need it again.

2. TYPESETTING MATH

2.1. **Some basics.** Math can be typeset two different ways:

(1) It can be typeset *inline*, such as this: $c^2 = a^2 + b^2 - 2ab\cos c$.
(2) It can be *displayed*, such as this:

$$c^2 = a^2 + b^2 - 2ab\cos c.$$

Lemma 2.1. *Some things look different when typset inline or displayed:* $\int_0^1 \frac{2x}{x^2+1}\,dx = \sum_{k=1}^\infty (-1)^{k-1}/k$, *but:*

$$\int_0^1 \frac{2x}{x^2+1}\,dx = \sum_{k=1}^\infty (-1)^{k-1}/k.$$

There are ways to override this: $\displaystyle\int_0^1 \frac{2x}{x^2+1}\,dx = \sum_{k=1}^\infty (-1)^{k-1}/k$ or $\int_0^1 \dfrac{2x}{x^2+1}\,dx = \sum_{k=1}^\infty (-1)^{k-1}/k$ or $\int_0^1 \frac{2x}{x^2+1}\,dx = \sum\limits_{k=1}^\infty (-1)^{k-1}/k$ or

$$\textstyle\int_0^1 \frac{2x}{x^2+1}\,dx = \sum_{k=1}^\infty (-1)^{k-1}/k$$

or

$$\int\limits_0^1 \tfrac{2x}{x^2+1}\,dx = \sum\nolimits_{k=1}^\infty (-1)^{k-1}/k.$$

Notice that LaTeX is good at making line breaks in prose but can sometimes get overwhelmed by line breaks in inline math.

Greek letters like $\alpha, \beta, \gamma, \dots$ are available, as well as those capitals $\Gamma, \Delta, \dots$ that don't just look like Roman letters. Lots of other symbols are defined as well—Google around to find them.

Remark. We analytic number theorists use certain notation like $\log(x+\sin x) = \log x + O(1)$ and $\log x \ll_\epsilon x^\epsilon$ and $(x-1)^3 \gg x^3$ and $\pi(x) \sim \operatorname{li}(x)$ and $\pi(x) - \operatorname{li}(x) = o\big(x(\log x)^{-A}\big)$ a lot.

In this course there are a lot of sums with multiple conditions of summation, so it's useful to know how to make operators like

$$\sum_{\substack{n\le x \\ n\equiv 3 \mod 4}} \quad \text{and} \quad \prod_{\substack{p>p_0 \\ p \text{ prime} \\ p\equiv 3 \pmod 4}} . \tag{1}$$

Proposition 2.2. *Subscripts, superscripts, and other doodads don't need brackets or spaces when they're a single symbol:*

$$\int_a^b \frac{8}{9} t_3^\pi \,dt = \int_a^b \frac{8}{9} t_3^\pi \,dt$$

But they do when they're more than one symbol:

$$\int_{3/4}^{2b} \frac{2x}{x^2+1} t_{odd}\,dt \ne \int_3 /4^2 b \frac{2}{x} x^2 + 1 t_o dd\,dt$$

You can make big parentheses and other delimiters by hand,

$$\Bigg(\bigg[\Big\{\big(4\big)\Big\}\bigg]\Bigg) + \Bigg(\bigg(\Big(\big(7\big)\Big)\bigg)\Bigg),$$

or you can let LaTeX do it for you:

$$\left(\sum_{n\le x}\left[\left\{\ell\right\}\left\{3^k\right\}x^{2^k}\right]\right) + \left(\prod_p \left[1 - 1/p^2\right\}\right.$$

2.2. **Displays with multiple lines.** If you have a very long *expression*, you can put it on multiple lines:

$$\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty}(k+1)x^k = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 8x^7$$
$$+ 9x^8 + 10x^9 + 11x^{10} + 12x^{11} + 13x^{12} + 14x^{13} + \cdots$$

If you have a series of *equations or inequalities*, you can put them on multiple lines and align them:

$$\begin{aligned} \frac{1}{(1-x)^2} &= \sum_{k=0}^{\infty}(k+1)x^k \\ &= 1 + \sum_{k=1}^{\infty}(k+1)x^k \\ &= 1 + 2x + \sum_{k=2}^{\infty}(k+1)x^k = \cdots \end{aligned}$$

You can split an expression by hand if it's necessary:

$$\begin{aligned} \frac{1}{(1-x)^2} &= \sum_{k=0}^{\infty}(k+1)x^k \\ &= 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + 8x^7 \\ &\qquad + 9x^8 + 10x^9 + 11x^{10} + 12x^{11} + 13x^{12} + 14x^{13} + \cdots \\ &= \left(\sum_{k=0}^{\infty} x^k\right)^2. \end{aligned}$$

3. AUTOMATIC NUMBERING

In my opinion, the most indispensable part of LaTeX is its ability to automatically number equations, theorems (and lemmas etc.), sections, page numbers, and bibliographic references—so that if you need to move material around while editing, the numerical references will automatically update themselves. Compare the `.tex` file to the output to see how this is done. Some examples already appeared back on page 1 in Section 1; here are some others.

Theorem 3.1. *Every even integer is followed by an odd integer.*

Proof. If n is an even integer, then set

$$m = n + 1; \tag{2}$$

then m is odd. □

Theorem 3.2. *Every odd integer is followed by an even integer.*

Proof. If n is an odd integer, and m is defined as in equation (2), then m is even. □

Remark. In Theorems 3.1 and 3.2, the word "followed" can be replaced by "preceded", although the proofs would need to be redone. See [2] for lots of facts not particularly related to this. You might also look at [1, page 217], although I have no idea what's there.

Corollary 3.3. *There are almost exactly as many even integers as odd integers between* 1 *and* x*.*

Some of the "funny lines" at the beginning of the `.tex` file control how theorems and their ilk are numbered: the system used in this document is for theorem numbers to start over in every section (2.1, 2.2, 2.3, then 3.1, etc.). Also, theorems, corollaries, propositions, and lemmas all use the same counter, so that after Lemma 2.1 comes Proposition 2.2, not Proposition 2.1. (This makes it easier for readers to find things.) *Equation* numbering, on the other hand, goes sequentially throughout the document (not resetting every section) on its own counter. All of these choices can be changed if you want, but I recommend the given settings.

4. MACROS

You can define your own macros to make repetitive phrases easier to type. For example, if you are reading this, you're probably taking MATH 613E: Topics in Analytic Number Theory. There are 7 students giving lectures, and also 7 students writing up notes from lectures. Everybody in the class is very ecstatic about this. Those macros were defined at the beginning of the `.tex` file. Another macro is re-defined right after this sentence in the `.tex` file.

If you're like me and don't like the way the `mod` and `pmod` commands work, as in equation (1), you can use the definition just before this sentence in the `.tex` file (which is complicated so that it will work correcly both inline and displayed). Now the `\mod` command displays like this:

$$\sum_{\substack{n\le x \\ n\equiv 3 \ (\mathrm{mod}\ 4)}} \tag{3}$$

You can put macro definitions and re-definitions anywhere in the `.tex` file, and they will be active from that point on. You can even put them inside sub-environments to make them temporarily in force if you really want to:

$$\alpha+\beta=\beta+\alpha$$

Lemma 4.1. $ALPHA+\beta=\beta+$ cx.

$1st\ Greek\ letter+\beta=\beta+\alpha$

$$\alpha+\beta=\beta+\alpha$$

That being said, for most purposes macros can simply be put at the top of the `.tex` file.

Remark. By this time, if you've been playing around with the `.tex` file, you might have gotten some errors while compiling (for example, you forget a closing bracket or something). The only helpful thing I can really say about that is that there are only a few types of errors that come up frequently, and you'll learn to figure out how to fix them. Occasionally you'll have to decipher why a correctly compiling file doesn't do what you think it should do; hopefully the above examples will help a bit with that. All answers are known, by someone!

5. SUGGESTED STRUCTURE OF YOUR ARTICLE

It's probably still a bit unclear exactly what you're supposed to produce for the written part of your course assessment: we've been using a lot of words—notes, expository article, etc.—that don't all match with one another. Here's my attempt at clarifying these expectations:

> You should produce an article, on the topic assigned to you, that could serve as a (short) chapter in a good textbook on the subject of the course. The article should contain everything that you would say in class, if you had all the time you wanted to lecture on the topic.

Some observations on this goal:

- The article should be written in complete sentences and paragraphs, with mathematics inserted where appropriate. It shouldn't be a series of equations with no explanation. See your favorite textbook for an example.
- The article shouldn't be a historical record of what the lecturer actually said. You'll have more time to write the article than the lecturer had to give the lectures, and so you'll be able to give a more complete and polished account.
- On the other hand, the article doesn't need to be written like a paper to be published in a research journal. Even if you had infinite time to lecture on the topic, that doesn't necessarily mean you have to include every single detail. Many intermediate results might be sufficiently covered in the prerequisite analytic number theory course, or in a topic selected by another lecturer. In fact, it might simply be that a particular technical lemma is so complicated that it would detract from the communication of the main topic. Don't use this as an excuse to leave out pertinent details, but it is an option where appropriate—just communicate clearly to the reader that this is what you've done.
- Include specific references to textbooks or published papers (supplemented, if appropriate, by excellent internet resources).

Here's a suggestion on how to organize your article (other outlines are also possible).

- Introduce the topic
  - Something the reader already knows that's related to the new topic
  - What new question do we want to answer?
  - Give context and history
  - What is conjectured?
- State the main result(s) (perhaps there has been a series of related results—some of them can be stated rigorously but without proving each and every one)
- Describe the overall strategy of the proof
- Break out some technical parts of the proof into preliminary lemmas and propositions (try to describe their role in the overall proof as you go)
- Give the proof(s) of the main result(s)
- Mention possible improvements and directions for future research
- Bibliography

The bottom line. If you want to run your ideas by me, get a reality check from me, or just ask for advice for a starting point, you are always welcome to ask me in class or in my office, or to email me.

REFERENCES

[1] E. Edelman, "The probability that a random real Gaussian matrix has k real eigenvalues, related distributions, and the circular law", *J. Multivariate Anal.* **60** (1997), no. 2, 202–232.

[2] H. L. Montgomery and R. C. Vaughan, *Multiplicative Number Theory I: Classical Theory*, Cambridge University Press (2007).

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