Today: Focus: iterated integrals in 2 variables

Notes:

- $\iint f(x,y)\,dx\,dy$ $\quad\iint f(x,y)\,dA$ doesn't make sense: need a **domain** of integration.

$$\iint_T f(x,y)\,dA \quad \text{or} \quad \int_a^b \int_{h(x)}^{g(x)} f(x,y)\,dy\,dx \quad \text{is fine.}$$

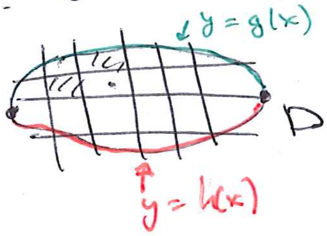
(The point: Need the limits: **domain** determines the limits!)

- Only doing "definite integrals". – There is no "indefinite integral" in 2 variables.

- The shape of the domain of integration is encoded in the limits.

- Iterated integral: **nested**: $\displaystyle\int_a^b \Big(\underbrace{\int_{h(x)}^{g(x)} f(x,y)\,dy}_{\text{pretend } x \text{ is a const.}} \Big)\,dx$

- **Note**: our definition was $\displaystyle\iint_D f(x,y)\,dA$ is defined as a limit of Riemann sums



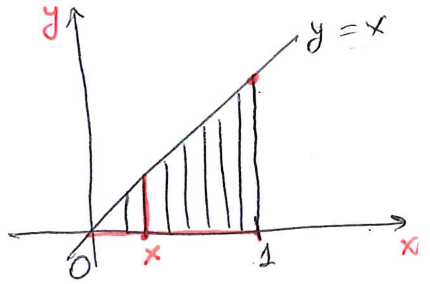
The only way to compute: make an iterated integral.

Worksheet 9

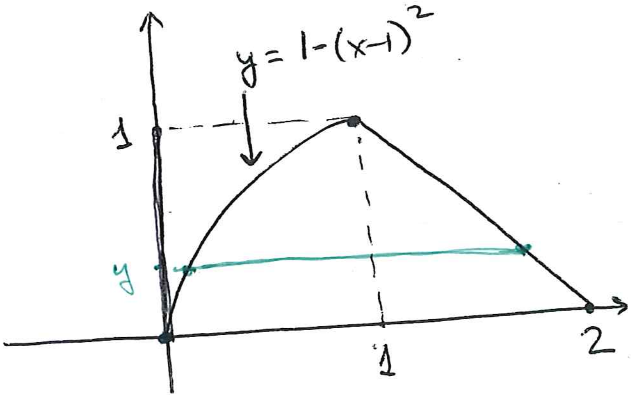
- Evaluate $\int_0^1 \underbrace{\int_0^x xy^2\,dy}_{} dx$, and draw the domain of integration.

$$\int_0^x xy^2\,dy = x\cdot\frac{y^3}{3}\Big|_0^x = x\cdot\frac{x^3}{3} \leftarrow \text{function of } x.$$

$$= \int_0^1 \frac{1}{3}\cdot x^4\,dx = \frac{1}{15}\cdot x^5\Big|_0^1 = \frac{1}{15}.$$



- Let $f(x,y)$ be a function. For the domain in the picture, write $\iint_D f(x,y)\,dA$ in two ways as an iterated integral.



$$\int_0^1 \int_{1\mp\sqrt{1-y}}^{2-y} f(x,y)\,dx\,dy$$

$$\iint f(x,y)\,dy\,dx$$

↑ see next page.

- on the outside integral limits have to be numbers!

Question 2:

1) $\int_0^1 \int_{\text{parabola}}^{\text{line}} f(x,y)\,dx\,dy$ (in our domain, $0 \le y \le 1$).

- when $y=0$, x goes from 0 to 2
- when $y=1$, $x=1$.
- for general y, x goes "from the parabola to the line".

! you can use y in the limits of integration inside the integral dy.
when y is fixed, $\int_y^2 dx$ is OK, but

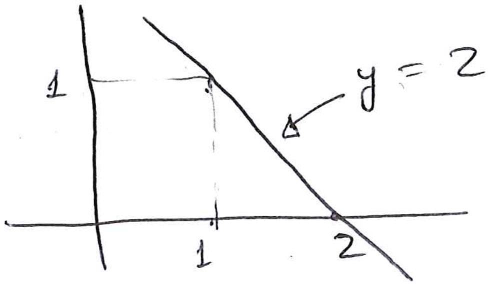
$\int_x^2 dx$ doesn't make sense!

- have to rewrite the equation of the parabola so that it gives x in terms of y.

$y = 1-(x-1)^2$ ← solve for x in terms of y.

$-(y-1) = (x-1)^2$, $x-1 = \pm\sqrt{1-y}$, $x = 1 \pm \sqrt{1-y}$.

- have to write the equation for our line:



$y = 2-x$, or $\boxed{x = 2-y}$.

$$\iint f(x,y)\,dy\,dx$$

want to say x goes from 0 to 2.

But: there is not one single function giving us the y-limit: when $0 \le x \le 1$, y goes up to the parabola.

but for $1 \le x \le 2$, it goes to the line!

So we need to break the integral into two:

$$\int_0^1 \int_0^{1-(x-1)^2} f(x,y)\,dy\,dx + \int_1^2 \int_0^{2-x} f(x,y)\,dy\,dx$$

At home, think of the following example

Example Evaluate $\displaystyle\int_{-1}^{0} \int_{-2}^{2x} \frac{e^y}{y}\,dy\,dx$.

Hint: there does NOT exist a formula for the antiderivative of the function $\frac{e^y}{y}$.