

## Table of integrals to remember.

### Power functions:

$$\int x^\alpha dx = \frac{1}{\alpha + 1} x^{\alpha+1} + C \quad \text{for all real numbers } \alpha \neq -1.$$
$$\int x^{-1} dx = \ln|x| + C.$$

### Rational functions:

For expressions of the form  $P(x)/Q(x)$ , where both  $P$  and  $Q$  are polynomials, if the degree of  $P$  is less than the degree of  $Q$ , use partial fractions.

If the degree of  $P$  is bigger or equal to the degree of  $Q$ , then do long division to get it to the form  $P(x) = Q(x)P_1(x) + R(x)$ , where  $R(x)$  has degree smaller than the degree of  $Q$ . Then you'll get:  $P(x)/Q(x) = P_1(x) + R(x)/Q(x)$ , and the first term  $P_1(x)$  is very easy to integrate, and for the second term  $R(x)/Q(x)$  you can use partial fractions.

### Exponential and logarithmic functions:

$$\int e^x dx = e^x + C.$$
$$\int a^x dx = \frac{1}{\ln a} a^x + C.$$
$$\int \ln(x) dx = x \ln x - x + C.$$

For expressions such as  $x^n \ln x$  or  $x^n e^x$ , use integration by parts.

### Trigonometric functions:

$$\int \sin(x) dx = -\cos(x) + C; \quad \int \cos(x) dx = \sin(x) + C.$$
$$\int \tan(x) dx = \ln|\sec(x)| + C = -\ln|\cos(x)| + C; \quad \int \cot(x) dx = \ln|\sin(x)| + C.$$
$$\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C; \quad \int \csc(x) dx = \ln|\csc(x) - \cot(x)| + C.$$
$$\int \sec^2(x) dx = \tan(x) + C \quad \int \csc(x)^2 dx = -\cot(x) + C.$$
$$\int \sec x \tan x dx = \sec(x) + C.$$

Note: the strategy for integrating  $\sin^n(x) \cos^m(x) dx$  is to make a substitution  $u = \cos(x)$  or  $u = \sin(x)$  (use the function whose power is odd); if both powers are even, use double-angle formulas to reduce to smaller powers of the functions of  $2x$ .

The strategy for integrating  $\tan^n(x) \sec^m(x)$  is: if  $m \geq 2$  is even, use the substitution  $u = \tan x$ . If  $n$  is odd, use  $u = \sec(x)$  (remember that  $\sec'(x) = \sec(x) \tan(x)$ ). In other cases, sometimes can use integration by parts to reduce to the integral of  $\sec(x)$  or  $\tan(x)$ , which you should remember (see the table above).

**Derivatives of the inverse trigonometric functions:**

$$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1}(x)$$

$$\int \frac{1}{1+x^2} = \tan^{-1}(x)$$

$$\int \frac{1}{x\sqrt{a^2-x^2}} = \frac{1}{a} \sec^{-1}(x/a) + C.$$

Note: if in these integrals, instead of  $1-x^2$  or  $1+x^2$  you have  $a^2-x^2$  or  $a^2+x^2$ , then use the substitution  $u = x/a$ ; if you have some other quadratic polynomial, then *complete the square*, and reduce it to  $a^2-x^2$ ,  $a^2+x^2$ , or  $x^2-a^2$ .

**Strategy for the other functions involving  $\sqrt{a^2-x^2}$  or  $a^2+x^2$ :** for  $\sqrt{a^2-x^2}$ , use the substitution  $x = a \sin(u)$ , for  $a^2+x^2$ , use the substitution  $x = a \tan(u)$ . If you have a quadratic polynomial inside a square root, then *complete the square* to reduce it to  $a^2-x^2$ ,  $x^2-a^2$ , or  $x^2+a^2$ .

**Always look for an easy way out first:** when computing definite integrals, do not forget to look for symmetry; always look for easy substitutions that could make the integral simpler. See if there is a clever integration by parts that could make it simpler, too.