1. Does there exist a continuous bijective function $f : \mathbb{R} \to \mathbb{R} - \{1\}$? Explain.
   Hint: Recall the Intermediate Value Theorem.

2. Let A_1, A_2, B_1, B_2 be non-empty sets such that $|A_i| = |B_i|$ for $i = 1, 2$. Prove that
   (a) $|A_1 \times A_2| = |B_1 \times B_2|$.

   (b) If $A_1 \cap A_2 = B_1 \cap B_2 = \emptyset$, then $|A_1 \cup A_2| = |B_1 \cup B_2|$.

   Remember — the sets may or may not be finite. This also applies to the remaining questions below.

3. Let A be an non-empty set. Prove that $|A| \leq |A \times A|$.

4. Prove that if A is a denumerable set, and there exists a surjective function from A to B (and B is infinite), then B is denumerable.

5. Prove that if A and B are denumerable sets, and C is a finite set, then $A \cup B \cup C$ is denumerable.
   Note: We use this fact, as well as the previous problem, when proving that the set of rational numbers $\mathbb{Q}$ is denumerable.

6. Prove that if a set A contains an uncountable subset, then A is uncountable.
   Note: We use this statement in the proof that the interval $(0, 1)$ is uncountable.

7. Let A be any uncountable set, and let $B \subset A$ be a countable subset of A. Prove that $|A| = |A - B|$.
   Hint. This is a generalization of one of the last problems from Workshop 5, and has a very similar solution.

8. (a) If $\mathcal{P}_{\text{fin}}(\mathbb{N})$ denotes the set of finite subsets of $\mathbb{N}$, show that $\mathcal{P}_{\text{fin}}(\mathbb{N})$ is denumerable.

   (b) If $\mathcal{P}_{\text{inf}}(\mathbb{N})$ denotes the set of infinite subsets of $\mathbb{N}$, show that $\mathcal{P}_{\text{inf}}(\mathbb{N})$ is uncountable.

   Hint: Use the previous problem.

9. Let A, B be sets. Prove that

   $$\text{if } |A - B| = |B - A| \text{ then } |A| = |B|.$$

   Hint: draw a careful picture.

10. Let $\{0, 1\}^{\mathbb{N}}$ be the set of all possible sequences of 0s and 1s. We have proved in class that this set is uncountable. Corollary 10.22 in the text states that in fact, the cardinality of this set is continuum: $|\mathbb{R}| = |\{0, 1\}^{\mathbb{N}}|$. Using this fact, prove that $|\mathbb{R} \times \mathbb{R}| = |\mathbb{R}|$.