

Review topics for the Math 223 midterm.

This is a rough list of the things you should definitely be able to do. I think the best way to use this review sheet is to try to recall or come up with an example for each concept or technique listed below. Then try to solve that example, and if you cannot (or could not come up with one to begin with), then re-read the corresponding section in the book. It is very important to try to think of all these concepts without opening the book, at first. Copies of this review sheet will not be allowed at the exam.

A good way to practice is to complete the “Test” sections in the book. You are responsible for everything tested in Sections 2.4, 3.3, 4.3, 5.4, 6.6, 7.4.

In addition, you are responsible for Section 2.5 (not covered by the “Test” sections), and Gaussian elimination (Section 7.5 but without column operations); reduced row echelon form (see notes for Feb. 13 on the website).

1. VECTOR SPACES, BASES, LINEAR DEPENDENCE AND INDEPENDENCE

- The notions of a vector space and a linear subspace. Determining whether a given subset of a vector space is a linear subspace.
- Deciding whether a given vector belongs to a given subspace.
- The notions of a basis and dimension.
- Deciding whether a given set of vectors is linearly independent, and find a subset that forms a basis of their linear hull.
- Finding bases of vector spaces.
- Fields: the notion of a field; the complex numbers; the field of p elements.

2. LINEAR TRANSFORMATIONS AND MATRICES

- The notion of a linear transformation, and how to associate a matrix with it (given a choice of a basis).
- An example: projector onto a linear subspace U along a subspace W .
- Rank of a linear transformation and rank of a matrix
- The kernel of a linear transformation
- The formula $\dim(\ker(A)) + \text{rk}(A) = \dim(V)$ (and need to understand why it holds).
- Finding the (reduced) row echelon form of a matrix via elementary row operations
- Using elementary operations to determine rank of a matrix

3. SYSTEMS OF LINEAR EQUATIONS

- Writing an (augmented) matrix associated with a system of linear equations, bring the matrix to RREF (reduced row echelon form), and how that gives a solution to the given system of equations.
- Pivot columns and pivot variables
- Finding a basis for the kernel of a linear transformation
- The inverse matrix; when does it exist?
- Finding the inverse of a matrix using elementary row operations.

4. DETERMINANTS

- Computing the determinant of a matrix using decomposition along one row or column.

- Cramer's rule.
- The adjugate matrix; the formula for an inverse of a 2×2 -matrix.
- Determinant of an upper-triangular matrix; determinant of a block-triangular matrix.
- Determinant of a product of matrices.