

Math 223, Homework 2: Vector spaces. Due Wednesday January 25.

1. Let V be a real vector space. Prove that:
 - (a) Prove that for $x \in V$, $0 \cdot x = \bar{0}$.
 - (b) Prove that $\lambda x = \bar{0}$ if and only if $x = \bar{0}$ or $\lambda = 0$.
 - (c) Prove that $(-1)x = -x$.
2. Problem 2.2 from Jänisch
3. Problem 2.3 from Jänisch
4. (a) Prove that all the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfying the differential equation
$$f''(x) - f(x) = 0$$
form a vector space.
 - (b) Prove that the solutions to this differential equation satisfying $f(\pi) = 0$ form its linear subspace.
5. Find all the complex numbers z such that $z^3 = 0$.
6. Prove that the set $F := \{a + b\sqrt{5} \mid a, b \in \mathbb{Q}\} \subset \mathbb{R}$ forms a field (with respect to the operations of addition and multiplication that it inherits from \mathbb{R}) (it is denoted by $\mathbb{Q}(\sqrt{5})$).