## Acknowledgement

UBC's Point Grey Campus is located on the traditional, ancestral, and unceded territory of the xʷməθkʷəy̓əm (Musqueam) people. The land it is situated on has always been a place of learning for the Musqueam people, who for millennia have passed on in their culture, history, and traditions from one generation to the next on this site.

## Course Information

| Course Title | Course Code Number | Credit Value |
|---|---|---|
| Linear Algebra (honours) | Math 223 | 3 |

**Lectures:** Mondays, Wednesdays and Fridays, 10-11am, in CHEM C 126.

**Textbook:**

- Klaus Jänisch, "Linear Algebra"; Undergraduate Texts in Mathematics, Springer-Verlag, 1994.
- Charles W. Curtis, "Linear Algebra, an introductory approach"; Undergraduate Texts in Mathematics, Springer-Verlag, 1994.

## Prerequisites

Official prerequisites: either (a) MATH 121 or (b) a score of 68% or higher in one of MATH 101, MATH 103, MATH 105, SCIE 001.

In fact, the course is pretty much self-contained and only uses high school material. We might use the concepts of differentiation/integration in some examples, but they are not essential. If you are in your first year and want to take the course, you can – apply to have the pre-requisites waived. However, the course does require some "mathematical maturity"; this explains the formal pre-requisites.

## Contact

Julia Gordon
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Office: MATH 217 Office hours: TBA, will be posted on the website.

## Website

`www.math.ubc.ca/~gor/Math223_fall23/math223f2023.html`

# Course description and Learning Outcomes

Linear algebra is, in some sense, the main tool available to humans (and their computers) for solving problems. One could say that it is a systematic study of the *systems of linear equations*. In many other mathematical subjects a central idea is to reduce complex phenomena to linear algebra (e.g., multivariable calculus describes local approximations of complicated functions of several variables by linear ones; the methods for solving differential equations are based on solving the companion system of linear equations, to name just a few). In other sciences and in all applications of mathematics, from theoretical physics, to weather forecasting, to AI and neural networks, linear algebra is the main tool as well as the *language* for describing the problems.

In short, a course in linear algebra is essential for anyone planning to do any work in science, engineering, or computer science, as well, as of course, for the future mathematicians. This course will emphasize *mathematical thinking* and proofs. Math 221 is more applications-oriented course, with more focus on computation. Throughout the course, we will emphasize the three main aspects (detailed below in the *Learning Objectives* section): **Computation, Linear Algebra as a language, and Mathematical thinking and proofs.**

**This course is aimed at excellent students; it is more abstract than Math 221 and covers more material.** It is possible to switch to Math 221 at any time during the term, but the earlier you do it, the easier the switch.

The course will consist of lectures, and a small computational component. You are strongly encouraged to be active on Piazza – asking questions and providing answers (and generally discussing the mathematics with your friends) is an excellent way to learn!

## *Learning Objectives*

**Basic Computational skills:**

- Vector and matrix arithmetic, including multiplication of matrices and finding inverses;
- Recognizing and solving systems of linear equations;
- Transforming matrices by column and row reduction;
- Deciding whether linear maps and matrices are invertible;
- Computing determinants by several techniques;
- Recognizing eigenvalue problems and computing eigenvalues and corresponding eigenvectors;

**More advanced Computational skills:**

- Deciding whether sets of vectors are dependent or independent, including computing spans and deciding membership in subspaces;
- Finding bases of vector spaces;
- Relating linear maps to matrices and vice versa;
- Computing in inner product spaces, including finding orthogonal complements.
- Being able to use some Python commands and Jupyter notebooks for the computation problems mentioned above.

**Basic Language skills:**

- Converting statements in mathematics and science to the language of linear algebra;
- Using the vocabulary of linear algebra to discuss these statements.

**Basic abstract algebra skills:**

- State the definitions and Theorems of the course;
- Prove simple statements involving the definitions;

**Abstract algebra skills:**

- Decide whether algebraic structures are vector spaces, and whether subsets are subspaces. Decide whether maps are linear transformations;
- Construct vector spaces from other vector spaces by various means;
- Recognize and use linear maps, their associated kernels and images;
- Prove statements about abstract sets.

**Metamathematical skills:**

- Solving problems where the method of solution has not been given in advance;
- Applying mathematical technique to convert solution ideas to rigorous solutions;
- Constructing proofs of given statements, especially in the context of linear algebra;
- Writing clear and concise proofs;
- Recognizing whether putative proofs are correct or incorrect.

## Course syllabus

*Course syllabus*

The course will cover approximately the whole textbook by Jänisch (with some sections omitted). We will cover one chapter every week. The main topics are:

**Vector spaces.**

- Preliminaries: sets and maps; Cartesian products of sets.
- Vector spaces - the definition.
- The field of complex numbers and other fields.
- Dimension of a linear space; Linear dependence and independence; bases.
- Linear subspaces.

**Matrices and determinants**

- Linear maps.
- Matrices.
- Quotient spaces.
- Rotations and reflections on the plane.
- Matrix multiplication.
- Elementary row operations; Gaussian elimination; rank of a matrix.
- Inverting matrices.
- A computational project in this section.
- Determinants.
- Systems of linear equations.

**More advanced topics**

- Euclidean spaces: inner products, orthogonal bases, orthogonal transformations.

- A note on groups of transformations.
- Eigenvalues and eigenvectors; Jordan canonical form of a matrix (without proof).
- The characteristic polynomial
- A computational project on eigenvalues and eigenvectors.

## Expectations

**You can expect from me:**

- To come prepared for class: knowing what we want to achieve, and how we will achieve it.
- Responses to your questions and concerns: continuously in class and during my office hours, within reasonable time on Piazza and within 48 hours by e-mail.
- Demanding homework and examinations.
- Clear explanations of what is correct in your work and what is not, and help in improving, including willingness to meet outside of class and discuss the material.

**You are expected:**

- To come prepared to class, having read relevant material and done problem sets.
- Working on the problem sets is absolutely essential for learning the material. **It is extremely rare for students who skip problem sets to do well on exams.**
- Actively participate in the course: read ahead of class, think about the material, and ask questions.
- Asking questions when you don't understand, or want to learn more, ensures that you get what you want out of the course. Ask me questions in class, on Piazza, by email, and during office hours. Also, ask your colleagues questions outside of class – you will both benefit from the discussion!
- To submit written work that is readable and communicates your ideas.
- To typeset your homework.
- To master the computational material on your own.

## Learning Activities and Assessments of Learning

Course mark will be based on the homework (20%), two computing projects (5% each), one "basics quiz" (10%) one midterm (15%) and the final exam (45%). The final exam will cover the entire course.

**Piazza** A great way to learn the material is to ask questions and generally discuss it with your peers. Please contribute to Piazza! If answering a question, please refrain from posting complete solutions. Five students who are the most active on Piazza will receive 1 point bonus added to their final mark.

# Official Policies

### General policies

- Late or missed exams and assignments will not be accepted for credit and will be given a grade of zero. In exceptional circumstances the missed work will be registered if you finish it and hand it in when you can.
- Registered homework will not count when the homework average is computed. The weight of a registered midterm will be transferred to the final exam.
- If you need to miss work / have missed work please let me know as soon as possible. Sometimes this means letting me know well in advance (example: you are scheduled to represent UBC in an athletic competition later in the term), but sometimes after the fact (example: you fall and break your arm, and write to me only after you are ready to resume schoolwork).
- In common situations (medical / conflicting responsibilities / compassionate grounds) no documentation is required for the first request for a concession. Instead, the student must submit the department's Academic Concession Form, available at `https://www.math.ubc.ca/Ugrad/ugradForm/StudentDeclaration_Academic_Concession_MATH.pdf`.
- First requests for concessions on other grounds, and subsequent requests for concessions must include documentation.
- Full details may be found in Senate Policy 135.

### Course-specific policies

- All assertions in your written work require proof unless the problem states otherwise. Regardless of the problem's operative word ("find", "solve", "establish", "calculate", "determine" ...), you must rigorously justify your answer.
- Written work should be presented carefully, with sufficient detail in complete English sentences. A "correct sequence of formulas" will only merit partial credit.
- I may designate material (calculational material or definitions, for example) for self-study, in which case you might be responsible for learning this material before it is used in class and in problem sets.
- You may not post homework or exam problems or any other course material to any website or discussion forum without coordinating with the instructor first. This includes without limitation Chegg, CourseHero, math.SE, r/math, Facebook, and the like.
- You will be asked to pledge to not post information or consult external sources for help. This is absolutely essential: the problems are difficult, but are chosen for pedagogical reasons. Evading the difficulty harms the students who do their best without such consultation.
- There will be up to eleven problem sets posted to the course website, due on Canvas at the beginning of class on the day shown on the schedule. I will drop the lowest score when calculating the homework grade.
- For the sanity of the grader, homework submissions must be typeset (not handwritten). Some suggestions for using TeX may be found on the course website.
- You are encouraged to work on solving the problems together. However, each of you must write your solutions independently, in your own words. You may (and should) share your ideas but you may not share your written work.
- It is possible that only certain problems from a problem set will be selected for grading.
- Solutions will be posted on the secure (Canvas) website.

### Final Exam Requirements

For a full description of the final exam regulations, see the UBC Calendar page on Student Conduct

during Examinations. In particular, notes, calculators, cell phones and other electronic devices are strictly prohibited from use during the exam. This includes use of cell phones for checking the time. The same regulations apply for the in-class tests.

## University Policies

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available on the `https://senate.ubc.ca/policies-resources-support-student-success`UBC Senate website.

## Learning Analytics

Learning analytics includes the collection and analysis of data about learners to improve teaching and learning. This course will be using the following learning technologies: Canvas and Piazza. Many of these tools capture data about your activity and provide information that can be used to improve the quality of teaching and learning. In this course, I plan to use analytics data to:

- View overall class progress
- Track your progress in order to provide you with personalized feedback
- Review statistics on course content being accessed to support improvements in the course
- Track participation in discussion forums
- Assess your participation in the course

Absence from Piazza will not affect your mark negatively; on the other hand, the top 5 contributors will be rewarded with 1 extra point.

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