Math 223, Basics Quiz.

NAME

Short answer section: In Questions 1-3 no proof is required but if you want you can add a line of explanation.

1. Decide if the following statements are True/False.

(a) Any vector space can be made into a field.
(b) If \( F \) is a field, it can be thought of as a 1-dimensional vector space over itself.
(c) Any vector space over \( \mathbb{C} \) can be thought of as a vector space over \( \mathbb{R} \).
(d) Any vector space over \( \mathbb{R} \) can be thought of as a vector space over \( \mathbb{C} \).
(e) If \( U_1, U_2 \) are linear subspaces of a vector space \( V \), then \( U_1 \cap U_2 \) is also a linear subspace.
(f) If \( U_1, U_2 \) are linear subspaces of a vector space \( V \), then \( U_1 \cup U_2 \) is also a linear subspace.
(g) \( \mathbb{R} \) (with its usual operations of addition and multiplication) is an infinite-dimensional vector space over \( \mathbb{Q} \).
(h) \( \mathbb{Q} \) (with its usual operations of addition and multiplication) is an infinite-dimensional vector space over the finite field \( \mathbb{F}_p \).
(i) In any vector space, vectors \( v_1, v_2 \) and \( v_3 = v_1 - v_2 \) are linearly dependent.
(j) In any vector space, any collection of vectors that contains \( \vec{0} \) is linearly dependent.
(k) Any vector space over \( \mathbb{R} \) contains at least three linearly independent vectors.

2. Find \((3 + 2i)^{-1}\) in \( \mathbb{C} \).

3. Describe all the linear subspaces of \( \mathbb{R}^3 \).

Proof Question: proof required.

4. Let \( V \) be an \( n \)-dimensional vector space over a field \( F \). Prove that a collection of \( n \) vectors \( \{v_1, \ldots, v_n\} \) forms a basis of \( V \) if and only if it is linearly independent.