Math 223, Basics Quiz.

NAME

Short answer section: In Questions 1-3 no proof is required but if you want you can add a line of explanation.

1. Decide if the following statements are True/False.

   (a) Any vector space can be made into a field.

   (b) If F is a field, it can be thought of as a 1-dimensional vector space over itself.

   (c) Any vector space over $\mathbb{C}$ can be thought of as a vector space over $\mathbb{R}$.

   (d) Any vector space over $\mathbb{R}$ can be thought of as a vector space over $\mathbb{C}$.

   (e) If U_1, U_2 are linear subspaces of a vector space V, then $U_1 \cap U_2$ is also a linear subspace.

   (f) If U_1, U_2 are linear subspaces of a vector space V, then $U_1 \cup U_2$ is also a linear subspace.

   (g) $\mathbb{R}$ (with its usual operations of addition and multiplication) is an infinite-dimensional vector space over $\mathbb{Q}$.

   (h) $\mathbb{Q}$ (with its usual operations of addition and multiplication) is an infinite-dimensional vector space over the finite field $\mathbb{F}_p$.

   (i) In any vector space, vectors v_1, v_2 and $v_3 = v_1 - v_2$ are linearly dependent.

   (j) In any vector space, any collection of vectors that contains $\overline{0}$ is linearly dependent.

   (k) Any vector space over $\mathbb{R}$ contains at least three linearly independent vectors.

2. Find $(3+2i)^{-1}$ in $\mathbb{C}$.

3. Describe all the linear subspaces of $\mathbb{R}^3$.

   Proof Question: proof required.

4. Let V be an n-dimensional vector space over a field F. Prove that a collection of n vectors $\{v_1, \dots, v_n\}$ forms a basis of V if and only if it is linearly independent.