Math 223, Homework 1: Sets and maps. Due Thursday September 14.

1. Let $A, B, C$ be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

2. Let $f : A \rightarrow B$ be a function.
   (a) Prove that for $A_1, A_2$ subsets of $A$, $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.
      Is the same statement true with $\cup$ replaced with $\cap$?
   (b) Prove that $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$ for $B_1, B_2$ -subsets of $B$.
      Is the same statement true with $\cup$ replaced with $\cap$?

3. Problem 1.1 from Jänisch

4. Problem 1.2 from Jänisch

5. Problem 1.3 from Jänisch

6. Let $A$ and $B$ be finite sets, and denote by $B^A$ the set of all functions from $A$ to $B$. Prove that $\#(B^A) = (\#B)^{\#A}$, where $\#A$ denotes the number of elements of $A$.

**Practice problems, not for handing in.**

7. Do the ”Test” for Chapter 1 (Section 1.3) in Jänisch.

8. Let $\mathcal{P}(A)$ be the set of all subsets of a set $A$ (it is called the power set of $A$).
   (a) Let $A = \{\emptyset, 1, \{1\}\}$. List all the elements of $\mathcal{P}(A)$.
   (b) For a subset $B$ of $A$, define the indicator function of $B$ by
       \[
       \chi_B(x) := \begin{cases} 
       1, & \text{if } x \in B \\
       0, & \text{if } x \notin B.
       \end{cases}
       
       Let $C = \{0, 1\}^A$ be the set of all functions from $A$ to the set $\{0, 1\}$. Define a bijective function from $\mathcal{P}(A)$ to $C$.
   (c) Prove that for any set $A$, $\#\mathcal{P}(A) = 2^{\#A}$. 